Quantifying Liquidity and Default Risks of Corporate Bonds over the Business Cycle*

Hui Chen   Rui Cui   Zhiguo He   Konstantin Milbradt

July 12, 2015

Abstract

By modeling debt rollover and endogenizing holding costs via collateralized financing, we develop a structural credit risk model to examine how the interactions between liquidity and default affect corporate bond pricing. The model captures realistic time variation in default risk premia and the default-liquidity spiral over the business cycle. Across different credit ratings, we simultaneously match the average default probabilities, credit spreads, and bid-ask spreads observed in the data. A structural decomposition reveals that the default-liquidity interactions account for 10∼24% of the observed credit spreads. We apply this framework to evaluate the liquidity-provision policies in the corporate bond market.

Keywords: Liquidity-Default Feedback, Rollover Risk, Over-The-Counter Markets, Endogenous Default

*Chen: MIT Sloan School of Management and NBER; e-mail: huichen@mit.edu. Cui: Booth School of Business, University of Chicago; e-mail: rcui@chicagobooth.edu. He: Booth School of Business, University of Chicago, and NBER; e-mail: zhiguo.he@chicagobooth.edu. Milbradt: Kellogg School of Management, Northwestern University, and NBER; email: milbradt@northwestern.edu. We thank Ron Anderson, Mark Carey, Pierre Collin-Dufresne, Thomas Dangl, Vyacheslav Fos, Joao Gomes, Lars Hansen, Jingzhi Huang, David Lando, Mads Stenbo Nielsen, Martin Schneider, and seminar participants at Chicago economic dynamics working group, ECB, Georgia Tech, Kellogg, Maryland, MIT Sloan, the NBER Summer Institute (Risk of Financial Institution, Capital Markets), Federal Reserve Board, AFA 2014, the USC Fixed Income Conference, University of Alberta, University of Calgary, Central European University, Wirtschaftsuniversität Wien, WFA, CICF, the Rothschild Caesarea Conference, Goethe Universität Frankfurt, and the Stanford Institute for Theoretical Economics Workshop for helpful comments. We acknowledge the financial supported from the Fama-Miller Center at Booth School of Business, University of Chicago.

1. Introduction

It is well known that a significant part of corporate bond pricing cannot be accounted for by default risk alone. For example, Longstaff, Mithal, and Neis (2005) estimate that "non-default components" account for about 50% of the spread between the yields of Aaa/Aa-rated corporate bonds and Treasuries, and about 30% of the spread for Baa-rated bonds. Furthermore, Longstaff, Mithal, and Neis (2005) find that non-default components of credit spreads are strongly related to measures of bond liquidity, which is consistent with evidence of illiquidity in secondary corporate bond markets (e.g., Edwards, Harris, and Piwowar (2007), Bao, Pan, and Wang (2011)).

Hitherto, the literature on structural credit risk modeling has almost exclusively focused on the "default component" of credit spreads. The "credit spread puzzle," as defined by Huang and Huang (2012), refers to the finding that, after matching the observed default and recovery rates, traditional structural models produce credit spreads for investment grade bonds that are significantly lower than those in the data. By introducing macroeconomic risks into structural credit models, Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) are able to explain the default components of the credit spreads for investment-grade bonds.[1] However, these models are silent on the sources of the non-default component of credit spreads and their potential interaction with the default component of the spreads.

This paper builds a tractable structural credit risk model that captures both the default and non-default components in corporate bond pricing by introducing secondary market search frictions (as in Duffie, Gârleanu, and Pedersen (2005)) into a model with aggregate macroeconomic fluctuations (e.g., Chen (2010)). Rather than simply piecing together the

[1] Chen (2010) relies on estimates of Longstaff, Mithal, and Neis (2005) to obtain the default component of the credit spread for Baa rated bonds, while Bhamra, Kuehn, and Strebulaev (2010) focus on the difference between Baa and Aaa rated bonds. The difference of spreads between Baa and Aaa rated bonds is supposed to take out the common liquidity component, a widely used practice in the literature. This treatment relies on the assumption that the liquidity components for bonds of different ratings are the same, which is at odds with the existing literature on the liquidity of corporate bonds, e.g., Edwards, Harris, and Piwowar (2007), Bao, Pan, and Wang (2011). Our paper relaxes this assumption by endogenously deriving the liquidity component for bonds with different ratings.

default and non-default components of credit spreads, our approach of jointly modeling default and liquidity risk over the business cycle is essential to understanding the interactions between these two components. Such interactions give rise to endogenous default as well as endogenous liquidity, which helps explain two general empirical patterns for the liquidity of corporate bonds: (1) corporate bonds with higher credit ratings tend to be more liquid; (2) corporate bonds are less liquid during economic downturns, especially for riskier bonds.[2] These interactions can also significantly raise the level of credit spreads and make them more volatile over the business cycle.

In our model, which builds on He and Milbradt (2014), investors face uninsurable idiosyncratic liquidity shocks, which drive up their costs of holding corporate bonds. Market illiquidity arises endogenously because to sell their bonds, investors have to search for dealers to intermediate transactions with other investors not yet hit by liquidity shocks, and in the meantime they incur costs of having to hold onto the bonds. The dealers set bid-ask spreads to capture part of the trading surplus. On the one hand, default risk crucially affects the trading surplus and thus the liquidity discount of corporate bonds. On the other hand, heavier liquidity discounts make it more costly for firms to refinance/roll-over their maturing debt, hence raising the probability of default. Thus, a *default-liquidity spiral* arises: when secondary market liquidity deteriorates, equity holders are more likely to default, which in turn worsens secondary bond market liquidity even further, and so on so forth.

Our model extends He and Milbradt (2014) in two important aspects. First, instead of assuming a constant exogenous holding cost due to liquidity shocks as in He and Milbradt (2014), we model holding costs that decrease with the firm's distance-to-default. These endogenous holding costs can be micro-founded based on collateralized financing. In this mechanism, investors hit by liquidity shocks raise cash either via cheaper collateralized financing (with the bond as collateral, subject to haircuts) or more expensive uncollateralized financing. When the firm gets closer to default, a lower bond price together with a larger haircut pushes investors toward more expensive uncollateralized financing, translating into

[2] See e.g., Edwards, Harris, and Piwowar (2007), Bao, Pan, and Wang (2011), Dick-Nielsen, Feldhütter, and Lando (2012), and Friewald, Jankowitsch, and Subrahmanyam (2012).

higher effective holding costs to investors.

Second, our paper introduces aggregate macroeconomics states as in Chen (2010) into He and Milbradt (2014) by explicitly modeling pro-cyclical liquidity together with cyclical variations in the firm's cash flows and aggregate risk prices. This allows us to generate significant time variation in *default risk premia*, an essential feature of the data. Moreover, as secondary market liquidity worsens (e.g., the meeting intensity with dealers decreases) in recessions when investors' marginal utilities are high, the model generates a *liquidity risk premium* in corporate bond prices. Together, these two types of risk premia magnify the quantitative effect of the default-liquidity spiral over the business cycle.

We follow the literature to set the pricing kernel parameters over two macroeconomic states (economic expansions and recessions) to fit a set of standard moments of asset prices. The parameters governing secondary bond market liquidity over macroeconomic states are either pre-fixed based on existing empirical studies and TRACE data (e.g., bond market turnover), or calibrated to match the moments that our model aims to explain (e.g., bid-ask spreads). We then apply our model with the same set of parameters to corporate bonds across four credit rating classes (Aaa/Aa, A, Baa, and Ba) and two different maturities (5 and 10 years). We make convexity adjustments within each rating class to account for firm heterogeneity within each class. Altogether, we only calibrate four free parameters, allowing ourselves only few degrees of freedom relative to the number of moments that our model tries to explain.

To evaluate model performance, we not only examine the cumulative default probabilities and credit spreads[3]—two common measures that the corporate bond pricing literature has primarily focused on—but also consider bid-ask spreads as an empirical measure of non-default risk in corporate bonds. Our paper differs from He and Milbradt (2014), whose calibration only matches the *unconditional* moments of credit spreads and bid-ask spreads, while we

[3]Empirically, credit spread is defined as the difference between the bond yield and Treasury yield. Over-the-counter search modeling mainly captures trading illiquidity of corporate bonds, but it is well-known that Treasuries enjoy other liquidity premia outside the search framework (e.g., Treasuries have the lowest hair cut in collateralized financing). To address this concern, we separate the risk-free rate and the Treasury yield by allowing for state-dependent Treasuries' liquidity premium, which is proxied by the repo-Treasury spreads observed in the data.

match *conditional* moments of credit spreads, bid-ask spreads and historic default rates as well as the empirical leverage distribution, across *both* aggregate states and rating classes.

In line with the structural credit literature, our calibration focuses predominantly on bonds with a 10-year maturity. The endogenous liquidity helps our calibrated model match closely the empirical patterns of default probabilities and credit spreads for 10-year bonds both cross-sectionally (across credit ratings) and over time (across the business cycle).[4] For non-default risk, the endogenous link between bond liquidity and a firm's distance-to-default allows us to generate the cross-sectional and business-cycle patterns observed in the data for bid-ask spreads. In contrast, a model with a perfectly liquid bond market implies a zero bid-ask spread. Overall, our model produces quantitatively reasonable non-default risk for corporate bonds. The model's performance on 5-year bonds is less satisfactory: our model features a term structure of credit spreads that is too steep relative to the data.

It is common practice in the empirical literature to decompose credit spreads into a liquidity and a default component, with the interpretation that these components are independent of each other (e.g., Longstaff, Mithal, and Neis (2005)). Our model suggests that both liquidity and default are inextricably linked. Such dynamic interactions are difficult to capture using reduced-form models with exogenously imposed default and liquidity risk components.[5]

To quantify the interaction between default and liquidity, we propose a structural decomposition of credit spreads that nests the common additive default-liquidity decomposition. Motivated by Longstaff, Mithal, and Neis (2005) who use CDS spread to proxy for default risk, we identify the "default" component in the total credit spreads of a corporate bond by pricing the same bond in a hypothetical perfectly liquid market but using the default threshold endogenously derived from the full model with liquidity frictions. After subtracting this "default" component, we identify the remaining credit spread as the "liquidity" component.

We further decompose the "default" component into a "pure default" and "liquidity-driven-default" component, and the "liquidity" component into a "pure liquidity" and "default-driven

[4]Our calibration on aggregate macroeconomic states focuses on normal expansions and recessions, but not crises. As a result, in constructing empirical moments for recessions, we exclude the recent financial crisis period from October (Q4) 2008 to March (Q1) 2009 throughout.

[5]See, e.g., Duffie and Singleton (1999) and Liu, Longstaff, and Mandell (2006).

liquidity" component. The "pure default" component is the spread for the same bond in a hypothetical setting with a perfectly liquid market as in Leland and Toft (1996) and hence equity holders optimally choose to default less often. The "pure liquidity" component is the spread for default-free bonds when there are over-the-counter search frictions as in Duffie, Gârleanu, and Pedersen (2005). The two interaction terms, i.e., the "liquidity-driven default" and the "default-driven liquidity" component, capture the endogenous positive spiral between default and liquidity. For instance, "liquidity-driven-default" reflects the rollover risk mechanism (He and Xiong (2012b)) in that firms relying on finite-maturity debt financing will default earlier when facing worsening secondary market liquidity.

Cross-sectionally, it is not surprising that the interaction terms are quantitatively more significant for bonds with lower ratings. We find that the interaction terms account for 10%$\sim$11% of the total credit spread of Aaa/Aa rated bonds and 17%$\sim$24% of the total spread of Ba rated bonds across the two aggregate states. We further perform a time-series default-liquidity decomposition for each rating category. Future studies can readily apply our decomposition scheme to individual firms.

By taking into account how individual firms' default decisions responds to changes in liquidity conditions, our model offers important new insight on evaluating the effectiveness of government policies that aim at improving market liquidity. Imagine a policy that improves the secondary market liquidity in a recession to the level of normal times. In our model, such a policy would lower the credit spread of Ba rated bonds in recession by about 102 bps, or 28% of the original spread. Furthermore, according to our decomposition, the policy's impact on the pure liquidity component only explains 42% of this reduction in credit spreads. In contrast, the liquidity-driven default component, which reflects the reduction in default risk when firms face smaller rollover losses, accounts for 9% of the reduction in the spread. The default-driven liquidity component, which captures the endogenous reduction of the liquidity premium for safer bonds, explains the remainder, about 49%. The prevailing view in the literature masks this interdependence between default and liquidity and thus could substantially under-estimate the impact of such liquidity policies.

The paper is structured as follows. Section 2 introduces the model, which is solved in Section 3. Section 4 presents the main calibration. Section 5 discusses the model-based default-liquidity decomposition and its applications. Section 6 concludes. All proofs are relegated to the Appendix.

2. The Model

Based on He and Milbradt (2014), we introduce secondary over-the-counter market search frictions (as in Duffie, Gârleanu, and Pedersen (2005)) into a structural credit risk Leland-type model with aggregate macroeconomic fluctuations. Our model extends He and Milbradt (2014) in two important aspects. First, we introduce aggregate states so that firm cash flows and secondary market liquidity are pro-cyclical while risk prices are countercyclical. Second, which is more important, in contrast to constant holding costs for investors hit by liquidity shocks as in He and Milbradt (2014), in our model the holding costs decrease with bond prices, a mechanism that is micro-founded via collateralized financing (see Appendix A).

2.1 Aggregate States and the Firm

The following model elements are similar to Chen (2010), except that we introduce bonds with an average finite maturity as in Leland (1998) so that rollover risk as in He and Xiong (2012b) is present.

2.1.1 Aggregate states and stochastic discount factor

The aggregate state of the economy is described by a continuous time Markov chain, with the current Markov state denoted by s_t and the physical transition density between state i

and state j denoted by $\zeta_{ij}^{\mathcal{P}}$. We assume an exogenous stochastic discount factor (SDF):[6]

$$\frac{d\Lambda_t}{\Lambda_t} = -r(s_t)dt - \eta\left(s_t\right)dZ_t^m + \sum_{s_t \neq s_{t-}} \left(e^{\kappa\left(s_{t-},s_t\right)} - 1\right) dM_t^{\left(s_{t-},s_t\right)}, \tag{1}$$

where Z_t^m is a standard Brownian Motion under the physical probability measure $\mathcal{P}$, $r\left(\cdot\right)$ is the risk-free rate, $\eta\left(\cdot\right)$ is the state-dependent price of risk for aggregate Brownian shocks, $dM_t^{(i,j)}$ is a compensated Poison process capturing switches between states i and j, and $\kappa\left(i,j\right)$ determines the jump risk premia such that the jump intensity between states i and j under the risk neutral measure $\mathcal{Q}$ is $\zeta_{ij}^{\mathcal{Q}} = e^{\kappa(i,j)}\zeta_{ij}^{\mathcal{P}}$. We focus on the case of binary aggregate states to capture the notion of economic expansions and recessions, i.e., $s_t \in \{G, B\}$. In the Internet Appendix we provide the general setup for the case of $n > 2$ aggregate states.

Later on, we will introduce undiversifiable idiosyncratic liquidity shocks to investors. Upon receiving a liquidity shock, an investor who cannot sell the bond will incur some holding costs. In the presence of such liquidity shocks, bond investors can still price assets using the SDF in (1) provided that the bond holdings only make up an infinitesimal part of the representative investor's portfolio.[7]

2.1.2 Firm cash flows and risk neutral measure

Consider a firm that generates cash flows at the rate of Y_t. Under the physical measure $\mathcal{P}$, the cash-flow rate Y_t dynamics, given the aggregate state s_t, follows

$$\frac{dY_t}{Y_t} = \mu_{\mathcal{P}}\left(s_t\right)dt + \sigma_m\left(s_t\right)dZ_t^m + \sigma_f dZ_t^f. \tag{2}$$

Here, dZ_t^m captures aggregate Brownian risk, while dZ_t^f captures idiosyncratic Brownian risk. Note that this is a nominal model as we do not specify a separate inflation process.

[6] We adopt a partial equilibrium approach where the pricing kernel is exogenous. For general equilibrium credit risk models with an endogenous pricing kernel, see, e.g., Gomes and Schmid (2010).

[7] Intuitively, if the representative agent's consumption pattern is not affected by the idiosyncratic shock (which is true if the bond holding is infinitesimal relative to the rest of the portfolio), then the representative agent's pricing kernel is independent of the idiosyncratic undiversified shocks.

Given the stochastic discount factor Λ_t, the cash-flow dynamics under the risk-neutral measure $\mathcal{Q}$ are

$$\frac{dY_t}{Y_t} = \mu_{\mathcal{Q}}(s_t)\,dt + \sigma(s_t)\,dZ_t^{\mathcal{Q}},$$

where $Z_t^{\mathcal{Q}}$ is a standard Brownian motion under $\mathcal{Q}$. The state-dependent risk-neutral growth rate and volatility of cash flows are given by

$$\mu_{\mathcal{Q}}(s_t) \equiv \mu_{\mathcal{P}}(s_t) - \sigma_m(s_t)\,\eta(s_t)\,, \text{ and } \sigma(s_t) \equiv \sqrt{\sigma_m^2(s_t) + \sigma_f^2}\,.$$

For the ease of notation, we work with log cash flows $y \equiv \log(Y)$ and denote the state dependence of the parameters using either a subscript or a superscript. Since we will work under the $\mathcal{Q}$-measure unless otherwise stated, we drop the subscript $\mathcal{Q}$ where no confusion can arise. For instance, the risk-neutral dynamics of log cash flows are simply given by

$$dy_t = \mu_s dt + \sigma_s dZ_t^{\mathcal{Q}}, \tag{3}$$

where the state-dependent drift and volatility are

$$\mu_s \equiv \mu_{\mathcal{Q}}(s_t) - \frac{1}{2}\sigma^2(s_t) = \mu_{\mathcal{P}}(s_t) - \sigma_m(s_t)\eta(s_t) - \frac{1}{2}\left[\sigma_m^2(s_t) + \sigma_f^2\right], \quad \sigma_s \equiv \sigma(s_t).$$

We obtain valuations for any asset by discounting the expected cash flows under the risk neutral measure $\mathcal{Q}$ with the risk-free rate. The unlevered firm value, given aggregate state s and cash-flow rate Y, is $v_U^s Y$, where the vector of price-dividend ratios $\mathbf{v}_U$ is

$$\mathbf{v}_U = \begin{bmatrix} r_G - \mu_G + \zeta_G & -\zeta_G \\ -\zeta_B & r_B - \mu_B + \zeta_B \end{bmatrix}^{-1} \mathbf{1}. \tag{4}$$

2.1.3 Firm's debt maturity structure and rollover frequency

The firm has a unit measure of bonds in place that are identical except for their time to maturity, with the aggregate and individual bond coupon and face value being c and p. As in Leland (1998), equity holders commit to keeping the aggregate coupon and outstanding face value constant before default, and thus issue new bonds of the same average maturity as the bonds maturing. The issuance of new bonds in the primary market incurs a proportional cost $\omega \in (0, 1)$. Each bond matures with intensity m, and the maturity event is i.i.d. across individual bonds. Thus, by law of large numbers over $[t, t + dt)$ the firm retires a fraction $m \cdot dt$ of its bonds. This implies an expected average debt maturity of $\frac{1}{m}$. The deeper implication of this assumption is that the firm adopts a "smooth" debt maturity structure with a refinancing/rollover frequency of m.[8]

2.2 Secondary Over-the-Counter Corporate Bond Market

We follow Duffie, Gârleanu, and Pedersen (2005) and He and Milbradt (2014) to model the over-the-counter corporate bond market. All trades have to be intermediated through dealers. Bond investors can hold either zero or one unit of the bond. They start in the H state without any holding cost when purchasing corporate bonds in the primary market. As time passes by, H-type bond holders are hit by idiosyncratic liquidity shocks with intensity ξ_s. These liquidity shocks lead them to become L-types who bear a positive holding cost hc_s per unit of time.

2.2.1 Endogenous Holding Cost

Different from He and Milbradt (2014) with no aggregate state switching and constant holding costs, we specify state-dependent holding costs $hc_s(P^s)$ that depend on the prevailing bond

[8] Most of the literature follows the tradition of Leland (1998) by assuming that the firm can fully commit to the financing policy with a constant aggregate debt face value and a constant maturity structure. For recent papers that relax this stringent assumptions, see Dangl and Zechner (2006), DeMarzo and He (2014), He and Milbradt (2015).

prices as follows:

$$hc_s(P^s) = \chi_s \left(N - P^s\right) \tag{5}$$

where $N > 0, \chi_G$ and χ_B are positive constants and $P^s\left(y\right)$ is the endogenous market price of the bond (to be derived in the next section) as a function of the log cash-flow y.

In Appendix A, we provide a micro-foundation for (5) based on collateralized financing. We interpret a liquidity shock as the urgent need for an investor to raise cash which exceeds the value of all the liquid assets that he holds, a common phenomenon for modern financial institutions. Bond investors first use their bond holdings as collateral to raise collateralized financing at the risk-free rate; and collateralized financing is subject to a haircut until they manage to sell the bonds. Any remaining gap must be financed through uncollateralized financing, which requires a higher interest rate. In this setting, the investor obtains less collateralized financing if (i) the current market price of the bond is lower, and/or (ii) the haircut for the bond is higher. In practice, (i) and (ii) often coincide, with the haircut increasing while the price goes down. The investor's effective holding cost is then given by the additional total uncollateralized financing cost, which increases when the bond price goes down. Under certain functional form assumptions on haircuts detailed in Appendix A, the holding cost takes the linear form in (5).

In Equation (5), if at issuance the bond is priced at par value p this implies a baseline holding cost of $\chi_s\left(N - p\right)$ (we will set $N > p$). With $\chi_s > 0$, the holding cost increases as the firm moves closer to default, so that the bond market value $P^s\left(y\right)$ goes down. This is the key channel through which our model captures the empirical pattern that lower rated bonds have worse secondary market liquidity.

The holding cost $hc_s(P^s)$ in (5) also depends on the aggregate state, through the following two channels. First, there is a direct effect, as we set $\chi_B > \chi_G$, which can be justified by the fact that the wedge between the collateralized and uncollateralized borrowing rates is higher in bad times. Second, there is an indirect effect, as the bond value $P^s\left(y\right)$ drops in bad times, giving rise to a higher holding cost for a given level of y. In Section 4.4 we study the quantitative importance of linking holding costs to firm's distant-to-default.

While we provide one micro-foundation for $hc_s(P^s)$ based on collateralized financing, there are other mechanisms via which institutional investors hit by liquidity shock incur extra losses if the market value of their bond holdings has dropped. For instance, suppose that corporate bond fund managers face some unexpected withdrawals when hit by a liquidity shock. As models with either learning managerial skills or coordination-driven runs would suggest, the deteriorating bond portfolios can trigger even greater fund outflows and extra liquidation costs.[9]

2.2.2 Dealers and Equilibrium Prices

There is a trading friction in moving the bonds from L-type sellers to H-type potential buyers currently not holding the bond, in that trades have to be intermediated by dealers in the over-the-counter market. Sellers meet dealers with intensity λ_s, which we interpret as the intermediation intensity of the bond market. For simplicity, we assume that after L-type investors sell their holdings, they exit the market forever, and that there is a sufficient supply of H-type buyers on the sideline.[10] The H-type buyers on the sideline currently not holding the bond also contact dealers with intensity λ_s. We follow Duffie, Gârleanu, and Pedersen (2007) to assume Nash-bargaining weights β for the investor and $1-\beta$ for the dealer, constant across all dealer-investor pairs and aggregate states.

Dealers use the competitive (and instantaneous) inter-dealer market to sell or buy bonds in order to keep a zero inventory position. When a contact between a L-type seller and a dealer occurs, the dealer can instantaneously sell the bond at the inter-dealer clearing price M to another dealer who is in contact with an H-type investor via the inter-dealer market. If a sale occurs, the bond travels from an L-type investor to an H-type investor with the help of the two dealers who are connected in the inter-dealer market.

For any aggregate state s, denote by D_l^s the bond value for an investor with type $l \in \{H, L\}$. Denote by B^s the bid price at which the L-type is selling his bond, by A^s the ask price at

[9]Models that analyze these issues include, for example, Berk and Green (2004), He and Xiong (2012a), Cheng and Milbradt (2012), and Suarez, Schroth, and Taylor (2014).

[10]This is an innocuous assumption made for exposition. Switching back from L to H is easily incorporated into the model. See the Appendix in He and Milbradt (2014) for details.

which the H-type is purchasing this bond, and by M^s the inter-dealer market price.

Following He and Milbradt (2014), we assume that the flow of H-type buyers contacting dealers is greater than the flow of L-type sellers contacting dealers; in other words, the secondary market is a *seller's market*. Then, Bertrand competition, the holding restriction, and excess demand from buyer-dealer pairs in the inter-dealer market drive the surplus of buyer-dealer pairs to zero.[11]

Proposition 1. *Fix valuations D_H^s and D_L^s. In equilibrium, the ask price A^s and inter-dealer market price M^s are equal to D_H^s, and the bid price is given by $B^s = \beta D_H^s + (1-\beta) D_L^s$. The dollar bid ask spread is given by $A^s - B^s = (1-\beta)(D_H^s - D_L^s)$.*

As is typical in any model with decentralized trading, there is no single "market price" in our over-the-counter market. However, we used the term "market price" when we modeled the endogenous holding cost in equation (5). Consistent with practice, we take the mid-price between the bid and ask prices to be the "market price", i.e.,

$$P^s = \frac{A^s + B^s}{2} = \frac{(1+\beta) D_H^s + (1-\beta) D_L^s}{2}. \tag{6}$$

Critically, the holding cost hc_s in (5) is linear in individual bond valuations, which allows us to derive bond values in closed-form later.

Finally, empirical studies focus on the proportional bid-ask spread, defined as the dollar bid-ask spread divided by the mid price:

$$ba_s(y,\tau) = \frac{2(1-\beta)(D_H^s - D_L^s)}{(1+\beta) D_H^s + (1-\beta) D_L^s}. \tag{7}$$

[11] Introducing a situation in which there is less buyers than potential sellers, i.e., a *buyer's market*, would entail tracking the value functions of investors on the sideline and significantly complicate the pricing of the deterministic maturity bonds we use for the calibration below. As it would not, however, add additional economic insights pertaining to credit risk in particular, we do not consider this case here.

2.3 Bankruptcy and Effective Recovery Rates

When the firm's cash flows deteriorate, equity holders are willing to repay the maturing debt holders only when the equity value is still positive, i.e. the option value of keeping the firm alive justifies absorbing current rollover losses and coupon payments. The firm defaults when its equity value drops to zero at some endogenous default threshold y^s_{def}, which is optimally chosen by equity holders. The bankruptcy costs is a fraction $1-\alpha$ of the value of the unleveled firm $v^s_U e^{y^s_{def}}$ at the time of default, where v^s_U is given in (4).

There is strong empirical evidence that bankruptcy recovery rates α depend on the aggregate state s (see Chen (2010)). Moreover, as emphasized in He and Milbradt (2014), if bankruptcy leads investors to receive the bankruptcy proceeds immediately, then bankruptcy confers a "liquidity" benefit similar to a maturing bond. This "expedited payment" benefit runs counter to the fact that in practice bankruptcy leads to the freezing of assets within the company and a delay in the payout of any cash depending on court proceeding.[12] Moreover, investors of defaulted bonds may face a much more illiquid secondary market (e.g., Jankowitsch, Nagler, and Subrahmanyam (2013)), and potentially a much higher holding cost once liquidity shocks hit due to regulatory or charter restrictions which prohibit certain institutions from holding defaulted bonds. These practical features, as shown in He and Milbradt (2014), lead to a type- and state-dependent bond recovery at the time of default:

$$\mathbf{D}^{def}(y) = \left[\alpha^G_H v^G_U, \alpha^G_L v^G_U, \alpha^B_H v^B_U, \alpha^G_L v^B_U\right]^T e^y. \tag{8}$$

Here, $\boldsymbol{\alpha} \equiv \left[\alpha^G_H, \alpha^G_L, \alpha^B_H, \alpha^B_L\right]^\top$ are the *effective* bankruptcy recovery rates at default. As explained in Section 4.1.3, when calibrating $\boldsymbol{\alpha}$, we rely exclusively on the market price of defaulted bonds observed immediately after default, and the associated empirical bid-ask spreads, to pin down $\boldsymbol{\alpha}$.

[12]For evidence on inefficient delay of bankruptcy resolution, see Gilson, John, and Lang (1990) and Ivashina, Smith, and Iverson (2013). The Lehman Brothers bankruptcy in September 2008 is a good case in point. After much legal uncertainty, payouts to the debt holders only started trickling out after about three and a half years.

2.4 Liquidity Premium of Treasury

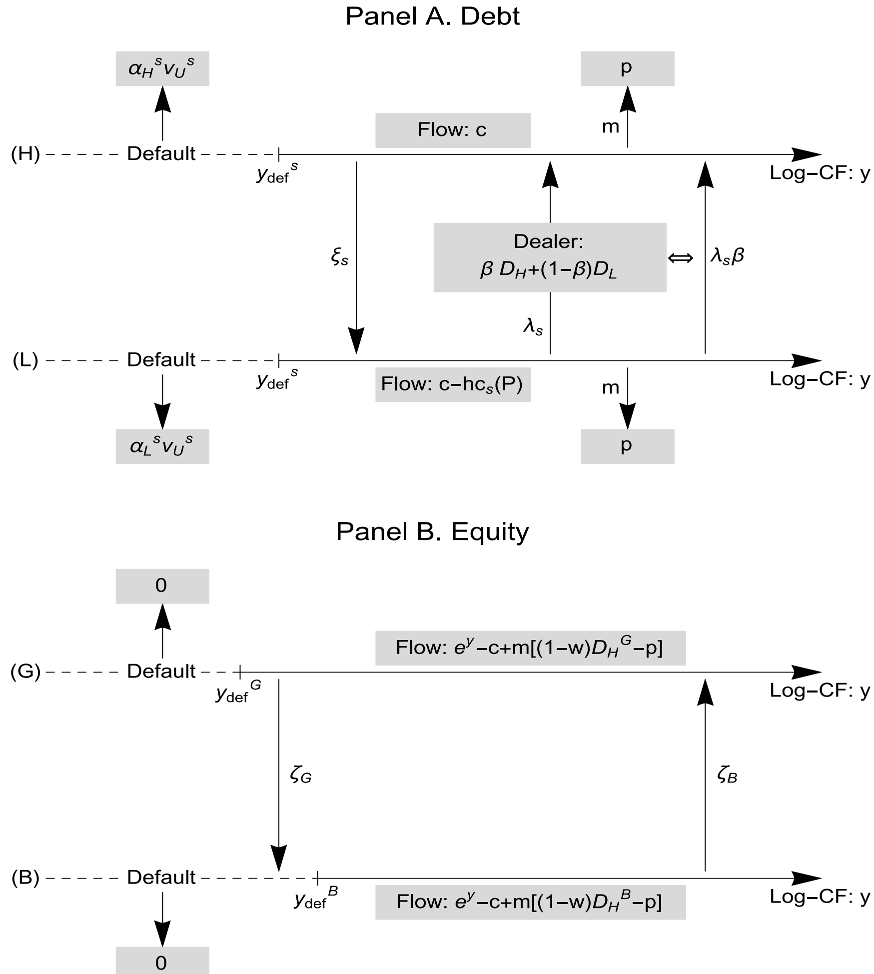
It has been widely recognized (e.g., Duffie (1996), Krishnamurthy (2002), Longstaff (2004)) that Treasuries, due to their special role in financial markets, are earning returns that are significantly lower than the risk-free rate, which in our model is represented by r_s in equation (1). The risk-free rate is the discount rate for future deterministic cash flows, whereas treasury yields also reflect the additional benefits of holding Treasuries relative to a generic default-free and easy-to-transact bond. The wedge between the two rates, which we term the "liquidity premium of Treasuries," represents the convenience yield that is specific to Treasury bonds, e.g., the ability to post Treasuries as collateral with a significantly lower haircut than other financial securities. Although this broad collateral-related effect is empirically relevant, our model is not designed to capture this economic force.

Nevertheless, our model can accommodate this effect by simply assuming that there are (exogenous) state-dependent liquidity premia Δ_s for Treasuries. Specifically, given the risk-free rate r_s in state s, the yield of Treasury bonds is simply $r_s - \Delta_s$. When calculating credit spreads of corporate bonds, following the convention we use the Treasury yield as the benchmark.

2.5 Summary of Setup

Figure 1 summarizes the cash flows to debt and equity holders. Panel A visualizes the cash flows to a debt holder in aggregate state s (excluding jumps of the aggregate state). The horizontal lines depict the current log cash flow y. The top half of the graph depicts an H-type debt holder who has not been hit by a liquidity shock yet. This bond holder receives a flow of coupon c each instant (all cash-flows in this figure are indicated by gray boxes). With intensity m, the bond matures and the investor receives the face value p. With intensity ξ_s the investor is hit by a liquidity shock and transitions to an L-type investor who receives cash flows net of holding costs of $(c - hc_s(P))\,dt$ each instant, where $P = [(1+\beta)\,D_H^s + (1-\beta)\,D_L^s]\,/2$ is the endogenous secondary market mid price. With intensity λ_s the L-type investor meets a dealer, sells the bond for $\beta D_H^s(y) + (1-\beta)\,D_L^s(y)$, and exits the market forever. To the

Figure 1: **Schematic graphic of cash flows to debt and equity holders**



debt holder, this is equivalent in value to losing the ability to trade but gaining a *recovery* intensity $\lambda_s\beta$ of transitioning back to being an H-type investor. Finally, when $y \le y^s_{def}$, the firm defaults immediately and the bond holders recover $\alpha^s_l v^s_U(y)$ depending on their type $l \in \{H, L\}$.

Panel B visualizes the cash flows to equity holders. The horizontal lines depict the current log cash flow y, where the top (bottom) line represents the aggregate G (B) state. Each instant, the equity holder receives a cash-flow $Y = e^y$ from the firm and pays the coupon c to debt holders. As debt is of finite average maturity, there are cash flows caused by continuously rolling over the debt: by the law of large numbers, a flow m of bonds come due each instant and each bond requires a principal repayment of p. At the same time, the firm reissues these maturing bonds with their original specification and raises an amount (after issuance costs) of $(1-\omega)D_H^s$ per bond depending on aggregate state $s \in \{G, B\}$. With intensity ζ_G the state switches from G to B and the primary bond market price decreases from $D_H^G(y)$ to $D_H^B(y)$, reflecting a higher default probability as well as a worsened liquidity in the market. In cases where $y \in \left(y_{def}^G, y_{def}^B\right)$ (as shown), the cash flows to equity holders are so low that they declare default immediately following the jump, receiving a payoff of 0.[13] Finally, with intensity ζ_B, the state jumps from B to G. Implicit in the model is that equity holders are raising new equity frictionlessly to cover negative cash flows before default.

Panel A and Panel B are connected via the primary market prices of newly issued bonds, i.e. $D_H^s(y)$ for $s \in \{G, B\}$. Although in the primary market the firm is able to locate and place newly issued bonds to H-type investors, the primary market prices of course reflect the secondary market illiquidity in Panel A, simply because forward-looking H-type investors take into account that they will face the illiquid secondary market in the future if hit by liquidity shocks. Through this channel, the secondary market illiquidity enters the firm's rollover cash flows in Panel B and affects the firm's default decision.

3. Model Solutions

For $l \in \{H, L\}$ and $s \in \{G, B\}$, denote by D_l^s the l-type bond value in aggregate state s, E^s the equity value in aggregate state s, and $\boldsymbol{y}_{def} = \left[y_{def}^G, y_{def}^B\right]^\top$ the vector of endogenous default boundaries. We derive the closed-form solution for debt and equity valuations as a function of y for given $\boldsymbol{y}_{def}$, along with the characterization of the endogenous default boundaries $\boldsymbol{y}_{def}$.

[13]Thus, the model features jumps to default even though the cash-flow process y is continuous.

3.1 Debt Valuations

Because equity holders default earlier in state B, i.e., $y^G_{def} < y^B_{def}$, the domains on which the bonds are "alive" change when the aggregate state switches. We deal with this issue by the method described below; see the Internet Appendix for the generalization of this analysis.

Define two intervals $I_1 = \left[y^G_{def}, y^B_{def}\right]$ and $I_2 = \left[y^B_{def}, \infty\right)$, and denote by $D^{s,i}_l$ the restriction of D^s_l to the interval I_i, i.e., $D^{s,i}_l(y) = D^s_l(y)$ for $y \in I_i$. Clearly, $D^{B,1}_l(y) = \alpha^B_l v^B_U e^y$ is in the "dead" state, so that the firm immediately defaults on interval I_1 when switching into state B (from state G). In light of this observation, on interval $I_2 = \left[y^B_{def}, \infty\right)$ all bond valuations denoted by $\mathbf{D}^{(2)} = \left[D^{G,2}_H, D^{G,2}_L, D^{B,2}_H, D^{B,2}_L\right]^\top$ are "alive."

Holding costs given liquidity shocks can be interpreted as negative dividends, which effectively lower the coupon flows that bond investors are receiving. Moreover, we directly apply the pricing kernel given in (1) without risk adjustments on the liquidity shocks. This treatment is justified by the assumption that the illiquid bond holding makes up only an infinitesimal part of the representative investor's portfolio. For further discussions, see footnote 7 and the end of Section 2.1.2.

Take $\mathbf{D}^{(2)}$ on interval I_2 as example. The bond valuation equation can be written in matrix form as follows:

$$\begin{aligned}\underbrace{\hat{\mathbf{R}} \cdot \mathbf{D}^{(2)}(y)}_{Discounting,4\times 1} &= \underbrace{\boldsymbol{\mu}}_{4\times 4}\underbrace{\left(\mathbf{D}^{(2)}\right)'(y)}_{4\times 1} + \frac{1}{2}\underbrace{\boldsymbol{\Sigma}}_{4\times 4}\underbrace{\left(\mathbf{D}^{(2)}\right)''(y)}_{4\times 1} + \underbrace{\hat{\mathbf{Q}} \cdot \mathbf{D}^{(2)}(y)}_{Transition,4\times 1} \\ &\quad + \underbrace{c\mathbf{1}_4}_{Coupon,4\times 1} + \underbrace{m\left[p\mathbf{1}_4 - \mathbf{D}^{(2)}(y)\right]}_{Maturity,4\times 1} - \underbrace{\left[\boldsymbol{\chi} \cdot \mathbf{N} - \boldsymbol{\chi} \cdot \mathbf{W} \cdot \mathbf{D}^{(2)}(y)\right]}_{Holding\,Cost,4\times 1}, \qquad (9)\end{aligned}$$

where $\hat{\mathbf{R}}, \boldsymbol{\mu}, \boldsymbol{\Sigma}, \hat{\mathbf{Q}}, \boldsymbol{\chi}$, and $\mathbf{W}$ are given in the Internet Appendix. Here, the left-hand-side is the required return for holding the bond. On the right-hand-side, the first three terms capture the evolution of cash-flows and the aggregate state. The fourth term is the coupon payment, the fifth captures the effect of debt maturing, and the last gives the holding costs. Key to solvability, from (5), is that endogenous debt valuations enter the holding cost linearly.

Proposition 2. *The bond values on interval i are given by*

$$\underbrace{\mathbf{D}^{(i)}}_{2i\times 1} = \underbrace{\mathbf{G}^{(i)}}_{2i\times 4i} \cdot \underbrace{\exp\left(\mathbf{\Gamma}^{(i)} y\right)}_{4i\times 4i} \cdot \underbrace{\mathbf{b}^{(i)}}_{4i\times 1} + \underbrace{\mathbf{k}_0^{(i)}}_{2i\times 1} + \underbrace{\mathbf{k}_1^{(i)}}_{2i\times 1} \exp(y), \tag{10}$$

where the matrices $\mathbf{G}^{(i)}$, $\mathbf{\Gamma}^{(i)}$ and the vectors $\mathbf{k}_0^{(i)}$, $\mathbf{k}_1^{(i)}$ and $\mathbf{b}^{(i)}$ are given in the Internet Appendix.

3.2 Equity Valuations and Default Boundaries

Recall that we assume that when the firm refinances its maturing bonds, it can place newly issued bonds with H investors in a competitive primary market subject to proportional issuance costs ω. This implies that there are rollover gains/losses of $m\left[\mathbf{S}^{(i)} \cdot \mathbf{D}^{(i)}(y) - p\mathbf{1}_i\right] dt$ as a mass $m \cdot dt$ of debt holders matures at each instant. Here, $\mathbf{S}^{(i)}$ is a $i \times 2i$ matrix that incorporates $\omega \in (0,1)$ as the proportional issuance cost in the primary corporate bond market, and selects the appropriate D_H.[14] Denote by double letters (e.g. $\mathbf{xx}$) a constant for equity that takes an analogous place to the single letter (i.e. $\mathbf{x}$) constant for debt, and we can write down the equity valuation equation on interval I_i. For instance, on interval I_2 we have

$$\begin{aligned}\underbrace{\mathbf{RR} \cdot \mathbf{E}^{(2)}(y)}_{Discounting, 2\times 1} \quad = \quad & \underbrace{\boldsymbol{\mu\mu}}_{2\times 2} \underbrace{\left(\mathbf{E}^{(2)}\right)'(y)}_{2\times 1} + \frac{1}{2} \underbrace{\mathbf{\Sigma\Sigma}}_{2\times 2} \underbrace{\left(\mathbf{E}^{(2)}\right)''(y)}_{2\times 1} + \underbrace{\mathbf{QQ} \cdot \mathbf{E}^{(2)}(y)}_{Transition, 2\times 1} \\ & + \underbrace{\mathbf{1}_2 \exp(y)}_{Cashflow, 2\times 1} - \underbrace{(1-\pi)\, c\mathbf{1}_2}_{Coupon, 2\times 1} + \underbrace{m\left[\mathbf{S}^{(2)} \cdot \mathbf{D}^{(2)}(y) - p\mathbf{1}_2\right]}_{Rollover, 2\times 1}\end{aligned} \tag{11}$$

where π is the marginal tax rate, and matrices $\boldsymbol{\mu\mu}$, $\mathbf{\Sigma\Sigma}$, $\mathbf{QQ}$, and $\mathbf{RR}$ are given in the Internet Appendix.

[14] Recall we assume that the firm issues to H-type investors in the primary market, which is consistent with our seller's market assumption in Section 2.2, i.e., there are sufficient H-type buyers waiting on the sidelines. For instance, for $y \in I_2 = [y_{def}(B), \infty)$ and state-independent issuance costs ω, we have $\mathbf{D}^{(2)} = \left[D_H^{G,2}, D_L^{G,2}, D_H^{B,2}, D_L^{B,2}\right]^\top$ and $\mathbf{S}^{(2)} = (1-\omega)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

Proposition 3. *The equity value is given by*

$$\underbrace{\mathbf{E}^{(i)}(y)}_{i\times 1} = \underbrace{\mathbf{GG}^{(i)}}_{i\times 2i} \cdot \underbrace{\exp\left(\boldsymbol{\Gamma\Gamma}^{(i)} y\right)}_{2i\times 2i} \cdot \underbrace{\mathbf{bb}^{(i)}}_{2i\times 1} + \underbrace{\mathbf{KK}^{(i)}}_{i\times 4i} \underbrace{\exp\left(\boldsymbol{\Gamma}^{(i)} y\right)}_{4i\times 4i} \underbrace{\mathbf{b}^{(i)}}_{4i\times i} + \underbrace{\mathbf{kk}_0^{(i)}}_{i\times 1} + \underbrace{\mathbf{kk}_1^{(i)}}_{i\times 1} \exp(y) \;\; \text{for } y \in I_i \tag{12}$$

where the matrices $\mathbf{GG}^{(i)}$, $\boldsymbol{\Gamma\Gamma}^{(i)}$, $\mathbf{KK}^{(i)}$, $\boldsymbol{\Gamma}^{(i)}$ *and the vectors* $\mathbf{kk}_0^{(i)}$, $\mathbf{kk}_1^{(i)}$ *and* $\mathbf{b}^{(i)}$ *are given in the Internet Appendix.*

Finally, the endogenous bankruptcy boundaries $\boldsymbol{y}_{def} = \left[y_{def}^G, y_{def}^B\right]^\top$ are given by the standard smooth-pasting condition:

$$\left(\mathbf{E}^{(1)}\right)'\left(y_{def}^G\right)_{[1]} = 0, \text{ and } \left(\mathbf{E}^{(2)}\right)'\left(y_{def}^B\right)_{[2]} = 0. \tag{13}$$

4. Calibration

4.1 Benchmark Parameters

We calibrate the parameters governing firm fundamentals and the pricing kernel to key moments of the aggregate economy and asset prices. Parameters governing time-varying liquidity conditions are calibrated to their empirical counterparts on bond turnover and observed bid-ask spreads.

[TABLE 1 ABOUT HERE]

4.1.1 SDF and cash flow parameters

In Table 1 we follow Chen, Xu, and Yang (2012) in calibrating the investors' pricing kernel and firm fundamentals. Start with the pricing kernel. To abstract from any term structure effects, we set the risk free rate $r_G = r_B = 5\%$ in both aggregate states. Transition intensities for the aggregate state give the average durations of expansions and recessions over the business cycle (10 years for expansions and 2 years for recessions). The price of risk η for

Brownian shocks and the jump risk premium $\exp(\kappa)$ are calibrated to match key asset pricing moments including the equity premium and price-dividend ratio (Chen (2010)).

Next, on the firm side, the cash-flow growth is matched to the average (nominal) growth rate of aggregate corporate profits. State-dependent systematic volatilities σ_m^s are calibrated to match the model-implied equity return volatilities with the data. We set the debt issuance cost ω in the primary corporate bond market to be 1% as in Chen (2010). Based on the empirical median debt maturity (including bank loans and public bonds reported in Chen, Xu, and Yang (2012)), we set $m = 0.2$ implying an average debt maturity of 5 years. The idiosyncratic volatility σ_f is chosen to match the average default probability of Baa firms. There is no state-dependence of σ_f as we do not have data counterparts for state-dependent Baa default probabilities. Finally, as explained later, the firm's current cash-flow level is chosen to match the empirical leverage in Compustat at the firm-quarter frequency.

Chen, Collin-Dufresne, and Goldstein (2009) argue that generating a reasonable equity Sharpe ratio is an important criterion for a model that tries to simultaneously match the default rates and credit spreads, as otherwise one can simply raise credit spreads by imposing unrealistically high systematic volatility and prices of risk. Our calibration implies an equity Sharpe ratio of 0.11 in state G and 0.20 in state B, which are close to the mean firm-level Sharpe ratio for the universe of CRSP firms (0.17) reported in Chen, Collin-Dufresne, and Goldstein (2009).

4.1.2 Secondary bond market illiquidity

We set the state-dependent liquidity premium Δ_s for Treasuries based on the average observed repo-Treasuries spread. This spread is measured as the difference between the 3-month general collateral repo rate and the 3-month Treasury rate. This is because the repo rate can be interpreted as the true "risk-free" rate, i.e., the discount rate for future deterministic cash flows. During the period from October 2005 to September 2013, the daily average of the repo-Treasury spread is 15 bps in the non-recession period and 40 bps in the recession period, leading us to set $\Delta_G = 15$ bps and $\Delta_B = 40$ bps.[15] These estimates are roughly consistent with the average liquidity premium reported in Longstaff (2004) based on Refcorp bond rates.

The liquidity parameters describing the secondary corporate bond market are less standard in the literature. We first fix the state-dependent intermediary meeting intensity based on anecdotal evidence, so that it takes a bond holder on average a week ($\lambda_G = 50$) in the good state and 2.6 weeks ($\lambda_B = 20$) in the bad state to find an intermediary to divest of all bond holdings. We interpret the lower λ in state B as a weakening of the financial system and its ability to intermediate trades. We then set bond holders bargaining power $\beta = 0.05$ independent of the aggregate state, based on empirical work that estimates search frictions in secondary corporate bond markets (Feldhütter (2012)).

We choose the intensity of liquidity shocks, ξ_s, based on observed bond turnover in the secondary market. In the TRACE sample from 2005 to 2012, the value-weighted turnover of corporate bonds during NBER expansion periods is about 0.7 times per year, which leads us to set $\xi_G = 0.7$. This is because given the relative high meeting intensities ($\lambda_G = 50$ and $\lambda_B = 20$), the turnover rate is almost entirely determined by the liquidity shock intensity ξ_s.[16] Although in the data there is no significant difference in bond turnover over the business cycle, in the baseline calibration we set $\xi_B = 1$ to capture the idea that during economic downturns institutional holders of corporate bonds are more likely to be hit by liquidity shocks. We provide comparative static results on ξ_s in Section 4.4.

The parameters χ_s's in equation (5) are central to determining the endogenous holding costs and thus the illiquidity of corporate bonds in the secondary market. As shown in Table 4, we calibrate $\chi_G = 0.06$ and $\chi_G = 0.11$ to target the bid-ask spreads for investment grade bonds in both aggregate states; and choose $N = 115$ (with a par bond value of $p = 100$; this choice of N ensures a strictly positive holding cost for all ratings) to roughly match the bid-ask spread for superior grade bonds in state G. In light of the particular micro-foundation in

[15] We exclude the crisis period from October 2008 to March 2009 (2008Q4 and 2009Q1) throughout the paper. Also, over a given horizon, the state-dependent instantaneous liquidity premium suggests that the average liquidity premium is horizon-dependent, but we ignore this effect for simplicity.

[16] The model implied expected turnover is $\frac{\xi_s \lambda_s}{\xi_s + \lambda_s} \simeq \xi_s$ when $\lambda_s \gg \xi_s$. Of course, we implicitly assume that all turnover in the secondary corporate bond market is driven by liquidity trades in our setting, while in practice investors trade corporate bonds for reasons other than liquidity shocks.

Section 2.2.1 where χ_s is interpreted as the wedge between collateralized and uncollateralized borrowing costs, both numbers are higher than the typical observed TED (LIBOR-Tbill) spreads. However, TED spreads (using the uncollateralized borrowing rate among a selected group of large and reputable banks) might underestimate the true cost of uncollateralized borrowing for typical firms. More importantly, as a quantitative paper, the state-dependent holding cost parameters χ_s in (5) are in reduced form and meant to capture factors beyond this particular micro-foundation of uncollateralized borrowing (see discussion at the end of Section 2.2.1).

4.1.3 Effective recovery rates

As explained in Section 2.3, our model features type- and state-dependent recovery rates α_l^s for $l \in \{L, H\}$ and $s \in \{G, B\}$. We first borrow from the existing structural credit risk literature, specifically Chen (2010), who treats the traded prices right after default as recovery rates, and estimates recovery rates of $57.55\% \cdot v_U^G$ in normal times and $30.60\% \cdot v_U^B$ in recessions (recall v_U^s is the unlevered firm value at state s).

Assuming that post-default prices are bid prices at which investors are selling, then Proposition 1 implies:

$$0.5755 \quad = \quad \alpha_L^G + \beta(\alpha_H^G - \alpha_L^G), \text{ and } 0.3060 = \alpha_L^B + \beta(\alpha_H^B - \alpha_L^B). \tag{14}$$

We need two more pieces of information on bid-ask spreads of defaulted bonds to pin down the α_l^s's. Edwards, Harris, and Piwowar (2007) report that in normal times (2003-2005), the transaction cost for defaulted bonds for median-sized trades is about $200bps$. To gauge the bid-ask spread for defaulted bonds during recessions, we take the following approach. Using TRACE, we first follow Bao, Pan, and Wang (2011) to calculate the implied bid-ask spreads for low rated bonds (C and below) for both non-recession and recession periods. We find that relative to the non-recession period, during recessions the implied bid-ask spread is higher by a factor of 3.1. Given a bid-ask spread of $200bps$ for defaulted bonds, this multiplier implies that the bid-ask spread for defaulted bonds during recessions is about $620bps$. Hence we have

$$2\% \quad = \quad \frac{2\left(1-\beta\right)\left(\alpha_H^G-\alpha_L^G\right)}{\alpha_L^G+\beta(\alpha_H^G-\alpha_L^G)+\alpha_H^G}, \text{ and } 6.2\% = \frac{2\left(1-\beta\right)\left(\alpha_H^B-\alpha_L^B\right)}{\alpha_L^B+\beta(\alpha_H^B-\alpha_L^B)+\alpha_H^B}. \tag{15}$$

Solving (14) and (15) gives us the estimates of:[17]

$$\boldsymbol{\alpha} = \left[\alpha_H^G = 0.5871, \alpha_L^G = 0.5749, \alpha_H^B = 0.3256, \alpha_L^B = 0.3050\right]. \tag{16}$$

These default recovery rates determine the bond recovery rate, a widely-used measure defined as the defaulted bond price divided by its promised face value. In our calibration, the implied bond recovery rate is 49.7% in state G and 24.5% in state B. The unconditional average recovery rate is 44.6%. These values are consistent with the average issuer-weighted bond recovery rate of 42% in Moody's recovery data over 1982-2012, and they capture the cyclical variations in recovery rates as documented in Chen (2010).

4.1.4 Degree of freedom in calibration

We summarize our calibration parameters in Table 1. Although there are a total of 28 parameters, most of them are "pre-fixed parameters " that are displayed in the upper part, Panel A, of Table 1. These are set either using the existing literature or based on moments other than the corporate bond pricing moments. We only freely pick (calibrate) four "calibrated parameters " displayed in the lower part, Panel B, of the table. They target the empirical moments that our model aims to explain: The idiosyncratic volatility σ_f is picked to target Baa firms' default probabilities; for holding cost parameters, N, χ_G, and χ_B are picked to target investment grade bid-ask spreads in both states and superior grade bid-ask spread in state G. As shown shortly, the number of degrees of freedom (4) is far below the number of our empirical moments that we aim to explain.

[17] This calculation assumes that bond transactions at default occur at the bid price. If we assume that transactions occur at the mid price, these estimates are $\alpha_H^G = 0.5813, \alpha_L^G = 0.5691, \alpha_H^B = 0.3140, \alpha_L^B = 0.2972$.

4.2 Empirical Moments

We consider four rating classes: Aaa/Aa , A, Baa, and Ba; the first three rating classes are investment grade, while Ba is speculative grade. We combine Aaa and Aa together because there are few observations for Aaa firms. We emphasize that previous calibration studies on corporate bonds focus on the *difference* between Baa and Aaa only, while we are aiming to explain the *level* of credit spreads across a wide range of rating classes. Furthermore, we report the model performance *conditional* on macroeconomic states, while typical existing literature only focus on *unconditional* model performance (Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010)). We classify each quarter as either in "state G" or "state B" based on NBER recessions. As the "B" state in our model only aims to capture normal recessions in business cycles, we exclude two quarters during the 2008 financial crisis, which are 2008Q4 and 2009Q1, to mitigate the effect caused by the unprecedented disruption in financial markets during crisis.[18]

[TABLE 2 ABOUT HERE]

4.2.1 Default probabilities

The default probabilities for 5-year and 10-year bonds in the data row of Panel A in Table 2 are taken from Exhibit 33 of Moody's annual report on corporate default and recovery rates (2012), which provides cumulative default probabilities over the period of 1920-2011. Unfortunately, state-dependent measures of default probabilities over the business cycle are unavailable.

4.2.2 Credit spreads

Our data of bond spreads is obtained using the Mergent Fixed Income Securities Database (FISD) trading prices from January 1994 to December 2004, and TRACE data from January

[18] For recent empirical research that focuses on the behavior of the corporate bond market during the 2007/08 crisis, see Dick-Nielsen, Feldhütter, and Lando (2012) and Friewald, Jankowitsch, and Subrahmanyam (2012).

2005 to June 2012. We follow the standard data cleaning process, e.g. excluding utility and financial firms.[19] For each transaction, we calculate the bond credit spread by taking the difference between the bond yield and the treasury yield with corresponding maturity. Within each rating class, we average these observations in each month to form a monthly time series of credit spreads for that rating. We then calculate the time-series average for each rating conditional on the macroeconomic state (whether the month is classified as a NBER recession), and provide the conditional standard deviation for the conditional mean. To account for the autocorrelation of these monthly series, we calculate the standard deviation using Newey-West procedure with 15 lags.

We report the conditional means of bond spreads for each rating class and their corresponding conditional standard deviations for both 5-year and 10-year bonds in the data row in Panel B of Table 2. In line with the existing literature, we focus on the model performance on the 10-year end. For example, Huang and Huang (2012) cover the period from the 1970's to the 1990's, and report an (unconditional) average credit spread of 63 bps for 10-year Aaa rated bonds, 91 bps for Aa, 123 for A, 194 for Baa, and 320 for Ba. Our unconditional 10-year average credit spreads which are the weighted average across conditional means reported in Panel B of Table 2 are fairly close to Huang and Huang (2012): 69 bps for Aaa/Aa, 102 bps for A, 169 for Baa, and 328 for Ba.

4.2.3 Bid-ask spreads

The non-default measure that we concentrate on is bid-ask spreads in the secondary market for corporate bonds, whose model counterpart is given in (7). Previous empirical studies have uncovered rich patterns of bid-ask spreads across aggregate states and rating classes. More specifically, we combine Edwards, Harris, and Piwowar (2007) and Bao, Pan, and Wang (2011) to construct the data counterparts for the bid-ask spread, as Edwards, Harris, and Piwowar (2007) only report the average bid-ask spread across ratings in non-recession times (2003-2005). The ratings considered in Edwards, Harris, and Piwowar (2007) are superior

[19]For FISD data, we follow Collin-Dufresne, Goldstein, and Martin (2001). For TRACE data, we follow Dick-Nielsen (2009).

grade (Aaa/Aa) with an bid-ask spread of 40 bps, investment grade (A/Baa) with an bid-ask spread of 50 bps, and junk grade (Ba and below) with a bid-ask spread of 70 bps.[20] For each grade, we then compute the measure of liquidity in Roll (1984) as in Bao, Pan, and Wang (2011), which we use to back out the bid-ask spread ratio between B-state and G-state. We multiply this ratio by the bid-ask spread estimated by Edwards, Harris, and Piwowar (2007) in normal times (2003-2005) to arrive at a bid-ask spread measure for the B state. These empirical estimates are reported in the data row of Table 4.

4.3 Calibration Results

4.3.1 Calibration method and convexity adjustment

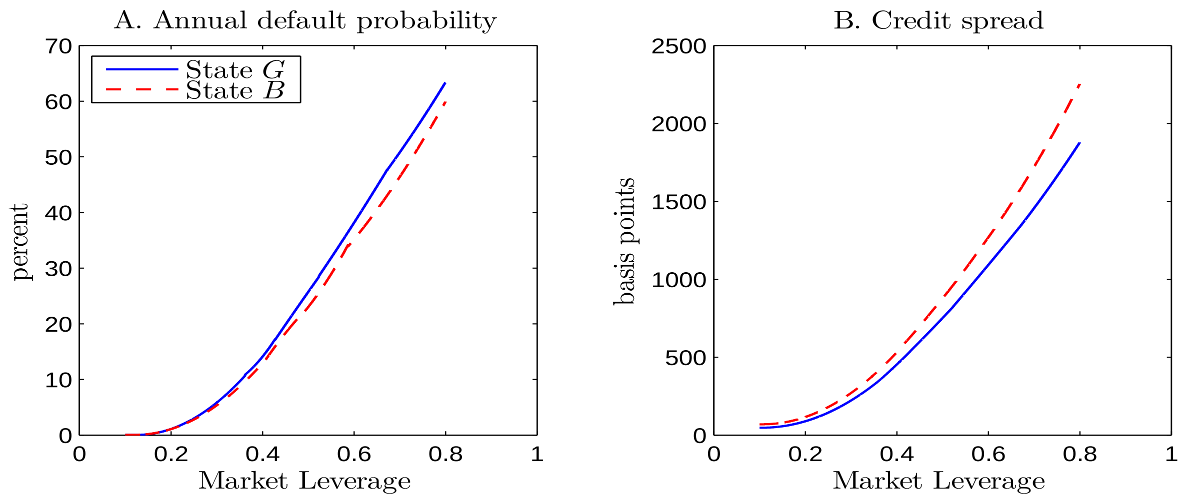
Any Compustat firm-quarter observation contains a firm's market leverage. We use the model to translate a firm's market leverage one-to-one into a given level of the log cash-flow y. We then compute the default probability and credit spread of bonds at 5 and 10 year maturity using Monte-Carlo methods firm-by-firm, and then aggregate within each rating category.[21] As is typical in structural corporate bond pricing models, we find that the model implied default probability and total credit spread are highly nonlinear in market leverage (see Figure 2).[22] The non-linearity inherent in the model implies, via Jensen's inequality, that the average credit spreads are higher than the spreads at average market leverage. This concern is empirically relevant, because the observed leverage distribution within each rating category is diverse as shown in Figure 3. We handle this convexity bias by following David (2008) in computing model implied aggregate moments: We compute the market leverage (i.e., book debt over the sum of market equity and book debt) of each Compustat firm (excluding

[20]We take the median size trade around 240K. Edwards, Harris, and Piwowar (2007) show that trade size is an important determinant for transaction costs of corporate bonds. But, for tractability reasons, we have abstracted away from trade size considerations.

[21]Recall that for tractability we assume that bonds have random maturity. In our calibration, we study bonds with deterministic maturities, which can be viewed as some infinitesimal bonds in the firm's aggregate debt structure analyzed in Section 2.1.3. Since the debt valuation derived in Proposition 2 does not apply, we rely on Monte-Carlo methods.

[22]The reason that default rate is higher in state G for the same market leverage is that the market leverage is higher in state B for the same book leverage due to the drop in equity value.

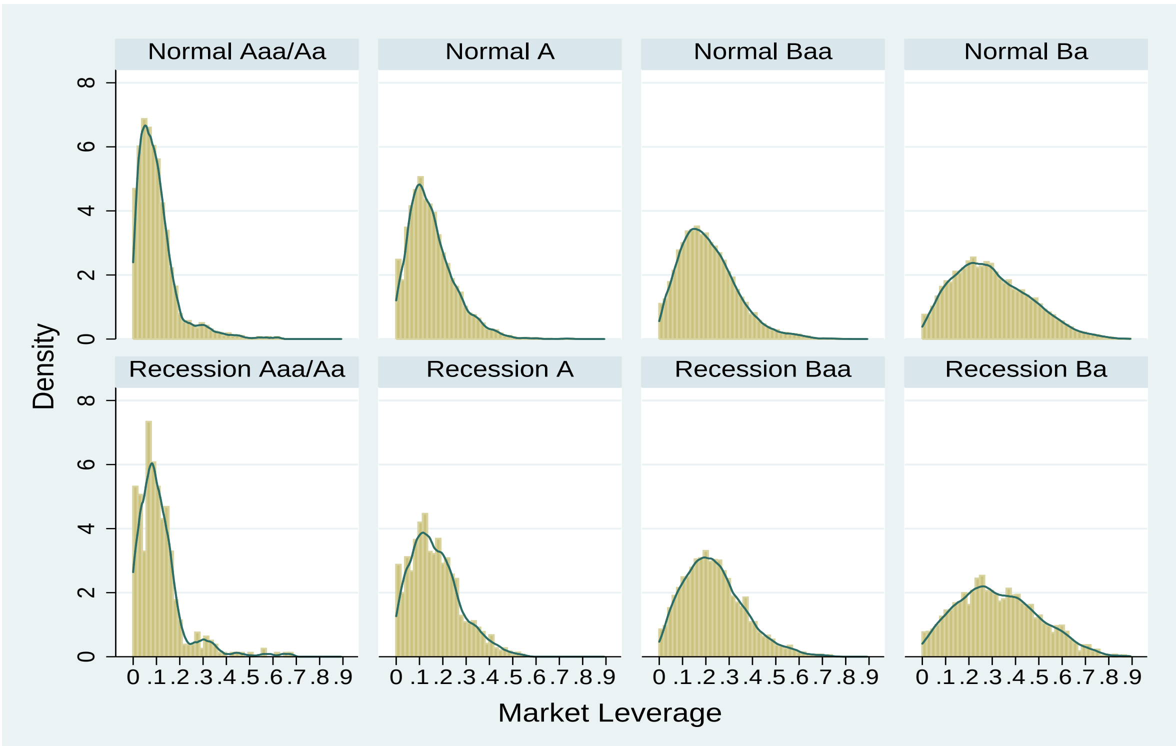
Figure 2: **Model implied default probability and credit spreads are nonlinear functions of the firm's market leverage.** The model parameters are given in Table 1.



financial and utility firms and other standard filters) that has rating information between 1994 and 2012. As described above, we then match the market leverage of each firm-quarter observation in Compustat to its model counterpart, which gives, for each observation, an implied current level of log cash-flow y. We then calculate the credit spread for each implied y and average across all firms according to the data implied density to get the average credit spread. We repeat the procedure for each rating class, each maturity, and each aggregate state. Hence, by construction, our methodology always exactly matches the empirical distribution of market leverage.

Relative to the existing literature, our calibration aims to explain the *level* of credit spreads across ratings, rather than the *difference* between ratings. For instance, Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010) focus on explaining the difference in spreads between Baa and Aaa rated bonds, which they consider the default component of Baa rated bonds under the assumption that the observed spreads for Aaa rated bonds are mostly driven by the liquidity premium. Because our framework endogenously models bond liquidity, we are able to match the total credit spreads that we observe in the data across rating classes, from superior ratings (Aaa/Aa) down to the high end of speculative rated bonds (here Ba).

Figure 3: **Empirical Distribution of Market Leverage for Compustat Firms by Aggregate State and Rating classes.** We compute quasi-market leverage for each firm-quarter observation in Compustat from 1994-2012, excluding financials, utilities, and firms with zero leverage. State B is classified as quarters for which at least two months are in NBER recession; the remaining quarters are G state. We exclude the financial crisis quarters 2008Q4 and 2009Q1.



Another important dimension that our paper improves upon over the existing literature is on the matching of *conditional* means of credit spreads. Because the success of Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010) hinges on the idea that the bond's payoff is lower in recessions with a higher marginal utility of consumption, checking whether the model implied bond spreads *conditional* on recessions match their empirical counterparts can be viewed as a disciplinary test for the mechanism proposed by those papers.

4.3.2 Default probabilities and credit spreads

Table 2 presents our calibration results on default probabilities (Panel A) and credit spreads across four rating classes (Panel B), for both 5-year and 10-year bonds.

Our calibration exercise puts more emphasis on the 10-year horizon.[23] The reason that we focus more on the 10-year—rather than the 5-year—horizon is that the structural credit literature has so far concentrated on 10-year spreads.

10-year default probabilities and credit spreads For 10-year bond maturities, our quantitative model is able to deliver decent matching of both cross-sectional and state-dependent patterns in default probabilities and credit spreads. Overall, relative to the data the model implied credit spread tends to overshoot in state G and undershoot in state B, but the match remains reasonable.

For Baa-rated bonds, our model gives a satisfactory match for 10-year credit spreads: in state G, the model predicts 182 bps while the data counterpart is 150 bps; in state B, we have 261 bps in the model versus 262 bps in the data. The match of default probabilities for Baa-rated bonds is also decent: the model predicts a 10-year cumulative default probability of 7.9% while it is 7% in the data.

More importantly, thanks to introducing liquidity into the structural corporate bond pricing model, we are able to produce reasonable credit spreads for Aaa/Aa bonds conditional on empirically observed default probabilities. The model implied default probability is 1.6%, slightly below the data counterpart of 2.1%. On credit spreads, Aaa/Aa rated bonds demand a credit spread of 86 (136) bps in state G (B), fairly close to the data counterpart of 61.2 (106) bps.

5-year default probabilities and credit spreads Previous studies (e.g., Huang and Huang (2012)) reveal that the class of structural models typically imply a much steeper term structure of credit spreads than reflected in the data, i.e., for relatively safe corporate bonds

[23] More specifically, within the reasonable range used in the literature, we have chosen the state-dependent risk price η and systematic volatility σ_m to deliver an overall good match for *10-year* Baa rated bonds.

(above Ba rated, say), the model-implied difference between 5-year and 10-year credit spreads is greater than its data counterpart. Our model suffers from the same issue; for instance, our model undershoots the 5-year Baa rated credit spreads (114 bps in the model versus 149 in the data in state G, and 191 bps in the model versus 275 in the data in state B). Certain interesting extensions of our model (e.g., introducing jumps in cash flows that are more likely to occur in state B) should help in this dimension, and we leave it to future research to address this issue.[24]

[TABLE 4 ABOUT HERE]

4.3.3 Bond market liquidity: bid-ask spreads

Our model features an illiquid secondary market for corporate bonds, which implies that the equilibrium credit spread must compensate the bond investors for bearing not only default risk but also liquidity risk. This new element allows us to investigate the model's quantitative performance on dimensions specific to bond market liquidity, here bid-ask spreads, in addition to default probabilities and credit spreads on which the previous literature has focused.

As explained in Section 4.2.3, Table 4 reports the empirical bid-ask spreads for bonds with different ratings across aggregate states. To calculate our model implied bid-ask spreads, again we correct for the convexity bias by relying on the empirical leverage distribution in Compustat of firms across ratings and aggregate states. Since the average maturity in TRACE data is around 8 years, the model implied bid-ask spread is calculated as the weighted average between the bid-ask spread of a 5-year bond and a 10-year bond.

Our model is able to generate both cross-sectional and state-dependent patterns that quantitatively match what we observe in the data, especially in normal times. As mentioned before, we calibrate two state-dependent holding cost parameters (χ_G and χ_B) to match the bid-ask spread of investment grade bonds over macroeconomic states. Also recall that we

[24] In unreported results, we find that the method of David (2008) which addresses the nonlinearity in the data (caused by the diverse distribution in leverage) has helped our model greatly to deliver a flatter term structure. This finding is consistent with Bhamra, Kuehn, and Strebulaev (2010). Nevertheless, this treatment is not strong enough to get the term structure right.

set $N = 115$ to roughly match the bid-ask spread of the superior bonds. The rest of bid-ask spreads are model implications, and we observe a satisfactory matching for the cross-sectional pattern of bond market illiquidity during recessions.[25]

4.4 Comparative Statics and Comparison to Alternative Models

Compared to earlier credit risk models with macroeconomic risks, such as Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010), our model adds an illiquid secondary market for corporate bonds. This section performs several comparative static exercises which allow us to assess the importance of secondary market (il)liquidity for the model implied credit spreads of corporate bonds. For better comparison to existing literature, in this section we focus on the calibrated 10-year credit spreads.

4.4.1 What if the state G illiquidity is the same as state B?

In our baseline calibration in Table 1, to capture the idea that institutional investors of corporate bonds are more likely to be hit by liquidity shocks (and hence sell their holdings) during economic downturns, we have set the state G liquidity shock intensity $\xi_G = 0.7$ which is smaller than the state B liquidity shock intensity $\xi_B = 1$. We now consider the case in which the bond investors are equally likely to experience a liquidity shock across both aggregate states, i.e. $\xi_G = \xi_B = 1$. We repeat the same procedure as before to generate the implied moments on default probabilities and credit spreads. The results are reported in the rows "$\xi_G = 1$" in Table 3; there, Panel A reports the default probabilities, while Panel B reports the credit spreads for each aggregate state.

Raising the state-G liquidity intensity ξ_G from 0.7 to 1 implies a worse state-G secondary market liquidity for corporate bonds, and it is not surprising to see that the implied credit spreads in state G across different ratings increase by about 10 bps against the baseline

[25] Although not reported here, the model-implied bid-ask spread of longer-maturity bonds is higher than that of shorter-maturity bonds, which is consistent with previous empirical studies (eg. Edwards, Harris, and Piwowar (2007); Bao, Pan, and Wang (2011)).

calibration. In our dynamic model, credit spreads in state B are also affected by the illiquidity in state G. Interestingly, we find that the state B credit spreads rise at a similar magnitude of about 10 bps, although much smaller in relative terms. We will see a similar result in Section 5.3.1 when we examine the policy which targets only the state-B secondary market liquidity. Finally, we observe that the implied default probabilities for all ratings go up, as worse secondary market liquidity leads firms to default earlier (and thus more frequently) due to the rollover risk channel.

4.4.2 What if the secondary market is perfectly liquid?

Let us now push the above exercise in the opposite direction by eliminating the secondary market illiquidity all together. By setting either the holding cost or the liquidity shock intensities to zero (i.e., either $\chi_s = 0$ or $\xi_s = 0$), we can see what our model calibration implies about default risk and credit spreads in the absence of liquidity frictions, which also helps isolate the effects of secondary market illiquidity.

The results for $\chi_s = 0$ (keeping other baseline parameters as in Table 1) are reported in the rows "$\chi_s = 0$" in Table 3. Obviously, the model implied bid-ask spreads are now identically zero. The credit spreads drop substantially, and more so for higher rated bonds. For highly rated Aaa/Aa firms, in state G the spread falls from 86.0 bps to 32.5 bps, while in state B it falls from 136 bps to 59.5 bps. Credit spreads for low-rated firms also fall, but by less in relative terms.

4.4.3 What if the holding cost does not depend on distance-to-default?

As emphasized previously, our paper differs from He and Milbradt (2014) in several dimensions. First, our model introduces time-varying Markov aggregate states. This work-horse setting in structural corporate bond pricing models proves to be successful in generating sizable and volatile credit risk premia which are important features of the data. As a result, our quantitative exercise matches the cross-sectional default probabilities and leverage distributions observed in the data, a task that He and Milbradt (2014) does not perform.

More importantly, in equation (5) the holding costs for investors hit by liquidity shocks are higher when they hold bonds that are closer to default. This generalization is key in generating significant cross-sectional difference in empirical bond liquidity. To isolate this effect, we consider another benchmark following He and Milbradt (2014) but with Markov aggregate states. More precisely, the alternative model, labeled by "hc_s," is the model considered in this paper, with holding costs hc_s that are state-dependent but no longer vary with the firm's distance-to-default.[26] The calibration takes the same baseline parameters, but chooses hc_s so that the implied bid-ask spreads for investment grade bonds in state $s = G, B$ match their empirical counterparts in the first row in Table 4. We then repeat the same procedure as before and report the model-implied default probabilities and credit spreads in the rows of "hc_s" in Table 3, and model implied bid-ask spreads in Table 4. The results in Table 4 show that, relative to our model, the "hc_s" model fails to deliver a sizable cross-sectional differences in bond illiquidity across different ratings.[27] The relatively rating-insensitive bond illiquidity of the "hc_s" model translates to a relatively flatter credit spreads across ratings compared to the our baseline model, as shown in Table 3. Together, these results highlight the importance of our assumption of (default) risk-sensitive holding costs in explaining the cross section of credit spreads and bond liquidity.

5. Structural Default-Liquidity Decomposition

Our structural model features an interaction between default and liquidity in determining the credit spreads of corporate bonds. This default-liquidity interaction is manifested in a lower default probability and credit spreads in the absence of liquidity frictions (see the rows "$\chi_s = 0$" in Table 3). One needs to take into account such interactions for a precise

[26]Recall that our baseline model has holding costs depend on bond price P, i.e., $hc_s(P)$, as in equation (5).

[27]*Qualitatively*, the endogenous default-illiquidity relation does not rely on the assumption of holding costs being decreasing in the firm's distance-to-default. As emphasized by He and Milbradt (2014) with constant holding cost, an endogenous default-illiquidity loop arises as long as bond investors face a worse liquidity in the post-default secondary bond market. In the calibration considered in He and Milbradt (2014), there is a reasonable cross-sectional difference despite a constant holding cost. Our results shown in Table 2 and Table 4 indicate the importance of matching default probabilities and leverage distributions in *quantitative* exercises.

default-liquidity decomposition of credit spreads.

It has been common practice in the empirical literature to decompose credit spreads into liquidity and default components in an additive way, such as in Longstaff, Mithal, and Neis (2005). From the perspective of our model, this "intuitively appealing" decomposition tends to over-simplify the role of liquidity in determining credit spreads. More importantly, the additive structure often leads to a misguided interpretation that liquidity and default are the causes of the corresponding name-sake components, and each component would give rise to the resulting credit spread if we were to shut down the other channel.

Which decomposition one uses has important implications for a range of policy related questions. For instance, as our proposed decomposition will highlight, part of the default risk comes from the illiquid secondary market. Thus, when policy makers are considering providing liquidity to the market, they should account for both the direct and indirect effect of this action: The direct effect on credit spreads comes from improving liquidity for a given amount of default risk, whereas the indirect effect stems from lowering the default risk via the rollover channel. The traditional perspective often overlooks this indirect effect, a quantitatively important effect according to our study.

5.1 Decomposition Scheme

We propose a structural decomposition that nests the additive default-liquidity decomposition common in the literature. For ease of illustration, we first isolate the exogenous liquidity premium of Treasuries. In other words, in this section we focus on studying the credit spread relative to the risk-free rate, a spread that consists of the default and liquidity parts. We then further decompose the default spread into a pure-default and a liquidity-driven-default part, and similarly decompose the liquidity spread into a pure-liquidity and a default-driven-liquidity part. More formally, denote the credit spread relative to the risk-free rate by $\hat{cs}_{rf}$, then we have

$$\hat{cs}_{rf} = \overbrace{\hat{cs}_{pureDEF} + \hat{cs}_{LIQ\to DEF}}^{\text{Default Component } \hat{cs}_{DEF}} + \overbrace{\hat{cs}_{pureLIQ} + \hat{cs}_{DEF\to LIQ}}^{\text{Liquidity Component } \hat{cs}_{LIQ}}. \tag{17}$$

We start by considering the decomposition of the spreads into a "Default" component and a "Liquidity" component. Imagine a hypothetical small investor who is not subject to liquidity frictions and consider the spread that this investor demands for the bond over the risk-free rate. The resulting spread, denoted by $\hat{cs}_{DEF}$, only prices the default event given the unchanged default boundaries y^s_{def}'s. Importantly, the default boundaries y^s_{def}'s in calculating $\hat{cs}_{DEF}$ are the ones that arise out of solving the model with liquidity frictions in equation (13).[28] Then, the "Liquidity" component is defined as the difference between the total spread $\widehat{cs}$ and the default component $\hat{cs}_{DEF}$, $\hat{cs}_{LIQ} \equiv \hat{cs} - \hat{cs}_{DEF}$. This two-way decomposition is roughly in line with the methodology of Longstaff, Mithal, and Neis (2005) who use the spreads of the relatively liquid CDS contract on the same firm to proxy for the default component in corporate bond spreads and attribute the residual to the liquidity component.

Next, we define the "Pure-Default" component $\hat{cs}_{pureDEF}$ as the spread implied by the benchmark Leland model without secondary market liquidity frictions (e.g., setting $\xi_s = 0$ or $\chi_s = 0$). Let $y^{Leland,s}_{def}$ denote the endogenous default boundaries in the absence of secondary market illiquidity. Because a perfectly liquid bond market leads to less rollover losses, equity holders default less often, i.e., $y^{Leland,s}_{def} < y^s_{def}$, implying a smaller pure-default component $\hat{cs}_{pureDEF}$ relative to the default component $\hat{cs}_{DEF}$. The difference $\hat{cs}_{DEF} - \hat{cs}_{pureDEF}$ gives the "Liquidity-driven Default" component, which quantifies the increase in default risk due to the illiquidity of the secondary bond market.

Following a treatment similar to that of the default component, we further decompose the liquidity component $\hat{cs}_{LIQ}$ into a "Pure-Liquidity" component and a "Default-driven Liquidity" component. Let $\hat{cs}_{pureLIQ}$ be the spread of a bond that is default-free but subject to liquidity frictions as in Duffie, Gârleanu, and Pedersen (2005).[29] The implied spread now captures the pure-liquidity component. The residual $\hat{cs}_{LIQ} - \hat{cs}_{pureLIQ}$ is what we term the "Default-driven Liquidity" part of our credit spread. Economically, when distance-to-default

[28] Hypothetically, this is the situation where all other bond investors are still facing liquidity frictions as modeled. Hence, the equity holders' default decision is not affected.

[29] Let p_{df} be this default-free price. Then the holding costs become a constant as we plug in $P^s = p_{df}$ in equation (5).

falls, lower bond prices give rise to higher holding costs, which contribute to the default-driven liquidity part.

This four-way decomposition scheme helps us separate *causes* from *consequences*, and emphasizes that lower liquidity (higher default risk) can lead to a rise in the credit spread via the default (liquidity) channel. Recognizing and further quantifying this endogenous interaction between liquidity and default is important in evaluating the economic consequence of policies that are either improving market liquidity (e.g., Term Auction Facilities or discount window loans) or alleviating default issues (e.g, direct bailouts). This decomposition has been proposed in He and Milbradt (2014), but we believe the quantitative implications are more trustworthy given the state-of-the-art calibration methodology in our paper.

5.2 Default-Liquidity Decomposition

5.2.1 Default-liquidity decomposition in cross-section

We perform the above default-liquidity decomposition for 10-year bonds. We follow the same procedure as in Section 4.3.1, i.e., we first identify the cash-flow state at the firm-quarter level based on the empirical leverage observed in Compustat, then aggregate over firms and quarters. The decomposition results for each aggregate state are presented in Table 5 and Figure 4, where credit spreads are reported relative to the risk-free rate.[30] For each component, we report its absolute level in bps, as well as the percentage contribution to the credit spread.[31]

[TABLE 5 ABOUT HERE]

As expected, the "pure default" component not only increases during recessions, but also rises for lower rated bonds. The fraction of credit spreads that can be explained by the "pure default" component starts from only 27% (23%) for Aaa/Aa rated bonds, and monotonically

[30]The state G (B) credit spread relative to the risk-free rate in Table 5 differs from the credit spread relative to Treasuries in Table 2 by the Treasury's liquidity premium $\Delta_G = 15$ bps ($\Delta_B = 40$ bps).

[31]In unreported results, we find that the decomposition results are largely unchanged when we vary the calibrated values of χ_s.

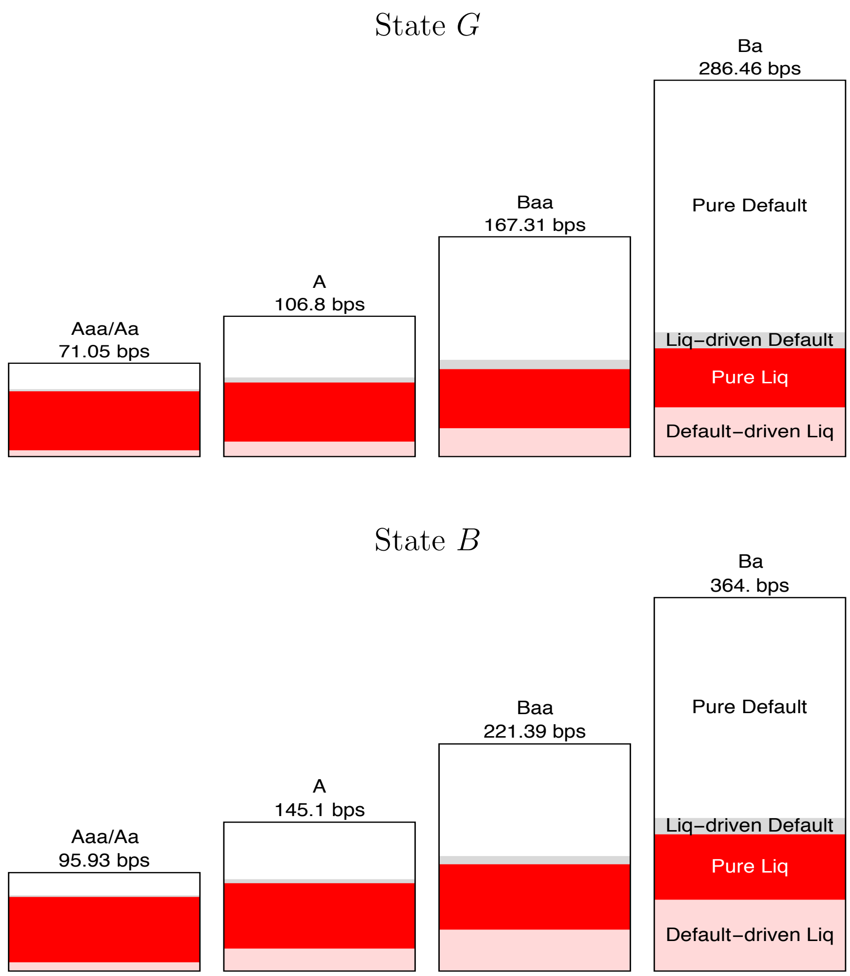
increases to about 67% (59%) for Ba rated bonds in state G (B). Not surprisingly, the "pure liquidity" component is higher in state B (63 bps) than state G (45 bps), but it does not vary across ratings. This is because the "pure liquidity" component captures the liquidity premium for a hypothetical default-free security whose holding costs are obviously independent of its rating but higher in state B.

The remainder of the observed credit spreads, which is around 10%∼17% (11%∼24%) in state G (B) depending on the rating, can be attributed to the novel interaction terms, i.e., either "liquidity-driven default" or "default-driven liquidity." The "liquidity-driven default" part captures how endogenous default decisions are affected by secondary market liquidity frictions via the rollover channel, which is quantitatively small for the highest rating firms (about 3% (2%) for Aaa/Aa rated bonds in state G (B)). As expected, its quantitative importance rises for low rating bonds: for Ba rated bonds, the liquidity-driven default accounts for about 4% (5%) in state G (B) of observed credit spreads.

The second interaction term, i.e., the "default-driven liquidity" component, captures how secondary market liquidity endogenously worsens when a bond is closer to default. Given a more illiquid secondary market for defaulted bonds, a lower distance-to-default leads to a worse secondary market liquidity because of the increased holding cost in (5). The "default-driven liquidity" component is quite significant across all ratings: it accounts for about 7∼13% (9∼19%) of the credit spread in state G (B) when we move from Aaa/Aa ratings to Ba ratings.

Comparison to He and Xiong (2012b) Motivated by the recent 2007/08 financial crisis during which financial firms experienced great turmoil, He and Xiong (2012b) highlight that the secondary bond market (il)liquidity in general pushes firms closer to default, i.e., the effect of "liquidity-driven default." There are at least two reasons why in our calibration the "liquidity-driven default" is quantitatively small relative to He and Xiong (2012b). First, our paper matches the empirical leverage distribution, which is important for a formal quantitative exercise. Second, our paper calibrates a longer debt maturity structure which implies a lower rollover risk. More precisely, He and Xiong (2012b) illustrate that the liquidity-driven-default

Figure 4: **Graphical illustrations of Structural Liquidity-Default Decomposition for 10-Year Bonds Across Ratings.** For numbers and explanations, see Table 5.



effect is significant for firms with a debt maturity structure of one-year, which is more relevant for financial firms. In contrast, our calibration focuses on non-financial firms whose average maturity structure is five-year. Indeed, the illustrating examples in He and Xiong (2012b) show that the liquidity-driven-default becomes much smaller for firms with an average debt maturity structure of six-year.

Comparison to Longstaff, Mithal, and Neis (2005) How do our decomposition results compare to those documented in Longstaff, Mithal, and Neis (2005)? Based on the corporate bond and CDS spreads, Longstaff, Mithal, and Neis (2005) estimate that a default component of about 51% of the credit spreads for 5-year Aaa/Aa rated bonds. For lower ratings, they report 56% for A, 71% for Baa, and 83% for Ba.

We mimic the above decomposition by computing the CDS spread in our model which is reported in the last column in Table 5; Appendix B explains how we calculate the model-implied CDS spread under the assumption of a perfectly liquid CDS market. Compared to Longstaff, Mithal, and Neis (2005), our model gives a somewhat smaller default component (defined as the ratio between CDS spread and credit spread). We have a default component of about 33% (28%) for Aaa/Aa rated bonds, 50% (46%) for A, 64% (58%) for Baa, and 79% (71%) for Ba in state G (B). Besides the difference in bond maturities (we focus on 10-year bonds while Longstaff, Mithal, and Neis (2005) study 5-year bonds), this discrepancy is mostly due to the fact that over our sample period, the empirical ratio between CDS spread and credit spreads is much lower.[32] A lower CDS spread naturally implies a smaller default component.

How about a higher intensity of liquidity shocks? The sample firms used in Longstaff, Mithal, and Neis (2005) are not the entire TRACE sample; they are firms with both bonds and CDS contracts, and in general have a higher turnover rate in the secondary bond market.

[32] In Longstaff, Mithal, and Neis (2005) whose sample period is from March 2001 to October 2002, the CDS spreads for Aaa/Aa rated bonds are about 59% of their corresponding credit spreads; 60% for A, 74% for Baa, and 87% for Ba. In contrast, in our data with a much longer sample period, these moments are 45% for Aaa/Aa, 52% for A, 51% for Baa, and 68% for Ba.

This difference highlights an important model parameter ξ (the intensity of liquidity shocks), and we find that the quantitative importance of interaction terms is sensitive to the choice of ξ's. In our baseline calibration, we set $\xi_G = 0.7$ and $\xi_B = 1$ to match the average secondary corporate bond market turnover rate in the entire TRACE sample. We could also choose ξ to match the bond market turnover rate for firms with both bonds and CDS contracts, which amounts to roughly doubling the implied liquidity shock intensities to $\xi_G = 1.4$ and $\xi_B = 2$. Appendix C illustrates that the interaction terms become significantly greater in this calibration.[33]

5.2.2 Default-liquidity decomposition in time-series

Now we apply the default-liquidity decomposition scheme in Section 5.1 to the time series of credit spreads. For a given credit rating, we use the observed leverage distribution of firms within each rating class for each quarter to compute the average credit spread and its four components in equation (17). We treat the NBER expansions and recessions as states G and B in our model, respectively. One caveat of this assumption is that the model treats the severity of the 2001 recession and the 2008-09 recession as the same (we have excluded 2008Q4 and 2009Q1 in this study so far), even though the latter was arguably more severe in reality.

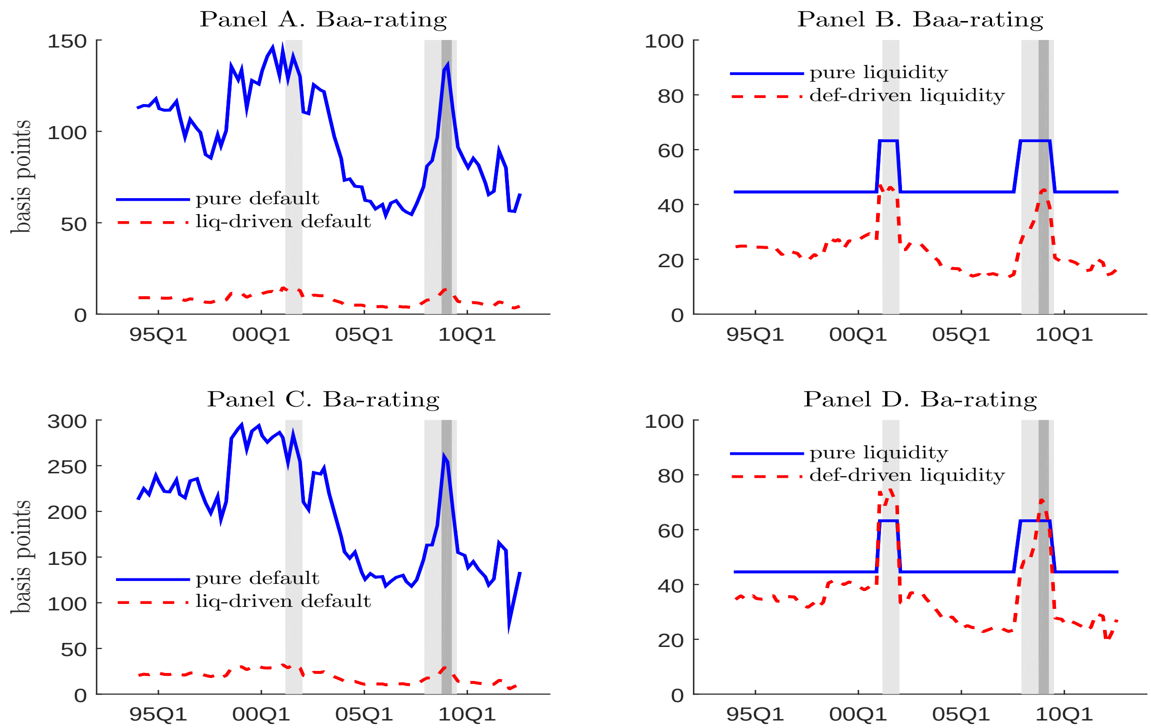
Figure 5 plots the time-series decomposition of credit spreads for Baa and B-rated bonds. To highlight the relative importance of the two interaction terms, in the left panels we plot the pure default spreads together with the liquidity-driven default spreads, while in the right panels we plot the pure liquidity spreads and the default-driven liquidity spreads.

The four components of the credit spreads are driven by both the time series variation in the leverage distribution and the aggregate state, with recessions identified by grey bars. Relative to Table 5, Figure 5 illustrates the time series variation in the cross-sectional leverage distribution.

Consider the default components in Panel A first. For both the Baa-rated and Ba-

[33]We have kept all other parameters in Table 1, except that we adjust the holding cost intercept down from $N = 115$ to $N = 110$ to deliver similar total credit spreads for Baa ratings.

Figure 5: **Time-series Structural Decomposition of Credit Spreads for Baa and Ba-rated Firms.** For each firm-quarter observation, we locate the corresponding cashflow level y that delivers the observed market leverage in Compustat (excluding financial and utility firms) and perform the structural liquidity-default decomposition for a 10-year bond following the procedure discussed in Section 4.3.1. For a given credit rating (Baa or Ba), we average across firms to obtain each component for each quarter from 1994 to 2012. Recessions are highlighted in grey. For completeness, we also calculate the model implied decomposition results for the crisis period from 2008Q4 to 2009Q1 in dark grey (which is excluded from the rest of this paper).



rated firms, the liquidity-driven default spreads have meaningful magnitudes, but they are significantly smaller than the pure default spreads. Not surprisingly, both default components rise in the two recessions in the sample. The model predicts that the pure default part for Baa spreads is lower in 2008-09 than in 2001. In reality, the credit spreads in 2008-09 recession were much higher than in the 2001 recession (especially in the financial crisis period from late 2008 to early 2009, which is marked in dark grey in the plots), potentially due to capital-deprived financial intermediaries around that time (He and Krishnamurthy (2013) and Chen, Joslin, and Ni (2014)). A more fine-tuned model with a “deep recession”—in addition to “normal recession” modeled here—would help on this front.

Moving on to liquidity components in Panel B, we observe that by definition the pure liquidity parts only depend on the aggregate state and are identical across ratings. In contrast, the default-driven liquidity spreads show significant variation over time and across ratings. For Baa-rated bonds, the default-driven liquidity spreads have a slightly lower magnitude than the pure liquidity spreads and similar time-series properties. For Ba-rated bonds, the default-driven liquidity spreads account for roughly half of the total liquidity spread on average, and for noticeably more in recessions.

As explained toward the end of Section 5.2.1, Figure 6 in Appendix C carries out the same exercise except that we double the implied liquidity shock intensities to $\xi_G = 1.4$ and $\xi_B = 2$. There, we see that the interaction terms become significantly larger, especially for the default-driven liquidity component.

5.3 Applications

5.3.1 Implications on evaluating liquidity provision policy

Our decomposition and its quantitative results are informative for evaluating policies that target lowering the borrowing cost of corporations in recession by injecting liquidity into the secondary market. As argued before, a full analysis of the effectiveness of such a policy should take account of the fact that firms' default policies respond to liquidity conditions and liquidity conditions respond to default risks. These endogenous forces are what our structural model is aiming to capture.

Suppose that the government is committed to launching certain liquidity enhancing programs (e.g., Term Auction Facilities or discount window loans) whenever the economy falls into a recession, envisioning that the improved funding environment for financial intermediaries alleviates the worsening liquidity in the secondary bond market. Suppose that the policy is effective in making the secondary market in state B as liquid as that of state G in terms of the characteristic liquidity parameters. More precisely, the policy helps increase the meeting intensity between L investors and dealers in state B, so that λ_B rises from 20 to $50 = \lambda_G$;

and reduce the state B holding cost parameter χ_B from 0.11 to $0.06 = \chi_G$.

In Table 6 we take the same cash flow distribution for each rating class and aggregate state as in Table 5, and calculate the credit spreads with and without the state-B liquidity provision policy on this fixed cash flow distribution. We find that a state-B liquidity provision policy lowers state-B credit spreads by about 52 bps for Aaa/Aa rated bonds and up to 102 bps for Ba rated bonds, which are about 54% and 28% of the corresponding credit spreads. Moreover, given the dynamic nature of our model, the state-B-only liquidity provision affects firms' borrowing costs in state G as well: the state-G credit spreads for Aaa/Aa (Ba) rated bonds go down by 29 (52) bps, or about 41% (18%) of the corresponding credit spreads.

Our structural decomposition further allows us to investigate the underlying driving force for the effectiveness of this liquidity provision policy. By definition, the "pure default" component remains unchanged given any policy that only affects the secondary market liquidity.[34] In Table 6, we observe that the pure-liquidity component accounts for about 83% (83%) of the drop in spread for Aaa/Aa rated bonds in state G (B). However, the quantitative importance of the pure-liquidity component diminishes significantly as we walk down the rating spectrum: for Ba rated bonds, it only accounts for about 46% (42%) in state G (B) of the decrease in the credit spread.

The market-wide liquidity provision not only reduces the investors' required compensation for bearing liquidity risk, but also alleviates some default risk. A better functioning financial market helps mitigate a firm's rollover risk and thus relaxes its default risk—this force is captured by the "liquidity-driven default" part. Table 6 shows that it accounts for around 5% (3%) of credit spread change in Aaa/Aa rated bonds, and goes up to 12% (9%) for lower Ba rated bonds in state G (B).

Given that the hypothetical policy was limited to only improving secondary market liquidity, the channel of "default-driven liquidity" is more intriguing. Such an interaction term only exists in our model with endogenous liquidity featuring a positive feedback loop between corporate default and secondary market liquidity. Interestingly, this interaction is

[34] Recall that the "pure default" component is defined by Leland (1998) which is independent of the secondary market liquidity.

more important quantitatively: it accounts for around 12% (14%) of credit spread change in Aaa/Aa rated bonds, and goes up to 42% (49%) for Ba rated bonds in state G (B).

[TABLE 6 ABOUT HERE]

5.3.2 Implications on accounting recognitions of credit-related losses

The interaction between liquidity and default as documented above has important implications for the ongoing debate regarding how accounting standards should recognize credit losses on financial assets. The interesting interplay between liquidity and default and their respective accounting recognitions have been illustrated in the collapse of Asset-Backed-Securities market during the second half of 2007. As Acharya, Schnabl, and Suarez (2013) document, because market participants are forward-looking, the liquidity problems (i.e., these conduits cannot roll over their short-term financing) occur before the actual credit-related losses (assets in the conduit start experiencing default). In a news release by Financial Accounting Standards Board (FASB) on 12/20/2013, the FASB Chairman Leslie F. Seidman noted that "the global financial crisis highlighted the need for improvements in the accounting for credit losses for loans and other debt instruments held as investment ... the FASB's proposed model would require more timely recognition of expected credit losses." However, there is no mentioning of the "liquidity" of these debt instrument at all. Our model not only suggests that (il)liquidity can affect the credit losses for these debt instruments, but more importantly offers a framework on how to evaluate the expected credit losses while taking into account the liquidity information.

6. Concluding Remarks

We build an over-the-counter search friction into a structural model of corporate bonds. In the model, default risk interacts with time varying macroeconomic and secondary market liquidity conditions. We calibrate the model to historical moments of default probability, bond yields, and empirical measures of bond liquidity. The model is able to match the conditional observed credit spreads across different rating classes and aggregate states. We propose a structural decomposition that captures the interaction of liquidity and default risks of corporate bonds over the business cycle and use this framework to evaluate the effects of liquidity provision policies during recessions. Our results identify quantitatively important economic forces that were previously overlooked in empirical researches on corporate bonds.

To focus on the interaction of liquidity and default, our model is cast in a partial equilibrium. Nevertheless, we believe these interactions have profound macroeconomic real impact, and the recent progress of general equilibrium models with credit risk (e.g., Gomes and Schmid (2010)) is the path for future research.

References

Acharya, V. V., P. Schnabl, and G. Suarez, 2013, "Securitization without risk transfer," Journal of Financial Economics, 107, 515–536.

Bao, J., J. Pan, and J. Wang, 2011, "The illiquidity of corporate bonds," The Journal of Finance, 66, 911–946.

Berk, J., and R. Green, 2004, "Mutual Fund Flows and Performance in Rational Markets," Journal of Political Economy, 112, 1269–1295.

Bhamra, H., L. Kuehn, and I. Strebulaev, 2010, "The levered equity risk premium and credit spreads: A unified framework," Review of Financial Studies, 23, 645–703.

Bongaerts, D., F. De Jong, and J. Driessen, 2011, "Derivative pricing with liquidity risk: Theory and evidence from the credit default swap market," Journal of Finance, 66, 203–240.

Chen, H., 2010, "Macroeconomic conditions and the puzzles of credit spreads and capital structure," The Journal of Finance, 65, 2171–2212.

Chen, H., S. Joslin, and S. X. Ni, 2014, "Demand for Crash Insurance, Intermediary Constraints, and Risk Premia in Financial Markets," Working Paper.

Chen, H., Y. Xu, and J. Yang, 2012, "Systematic Risk, Debt Maturity, and the Term Structure of Credit Spreads," Discussion paper, National Bureau of Economic Research.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein, 2009, "On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle," Review of Financial Studies, 22, 3367–3409.

Cheng, I., and K. Milbradt, 2012, "The Hazards of Debt: Rollover Freezes, Incentives, and Bailouts," Review of Financial Studies, 25, 1070–1110.

Collin-Dufresne, P., R. S. Goldstein, and S. J. Martin, 2001, "The Determinants of Credit Spread Changes," Journal of Finance, 56, 2177–2207.

Dangl, T., and J. Zechner, 2006, "Debt maturity and the dynamics of leverage," Working Paper.

David, A., 2008, "Inflation uncertainty, asset valuations, and the credit spreads puzzle," Review of Financial Studies, 21, 2487–2534.

DeMarzo, P., and Z. He, 2014, "Leverage dynamics without commitment," Working Paper.

Dick-Nielsen, J., 2009, "Liquidity Biases in TRACE," Journal of Fixed Income, 19, 43–55.

Dick-Nielsen, J., P. Feldhütter, and D. Lando, 2012, "Corporate bond liquidity before and after the onset of the subprime crisis," Journal of Financial Economics, 103, 471–492.

Duffie, D., 1996, "Special Repo Rates," Journal of Finance, 51, 493–526.

Duffie, D., N. Gârleanu, and L. Pedersen, 2005, "Over-the-Counter Markets," Econometrica, 73, 1815–1847.

Duffie, D., N. Gârleanu, and L. H. Pedersen, 2007, "Valuation in Over-the-Counter Markets," Review of Financial Studies, 20, 1865–1900.

Duffie, D., and K. J. Singleton, 1999, "Modeling Term Structures of Defaultable Bonds," Review of Financial Studies, 12, 687–720.

Edwards, A., L. Harris, and M. Piwowar, 2007, "Corporate bond market transaction costs and transparency," The Journal of Finance, 62, 1421–1451.

Feldhütter, P., 2012, "The same bond at different prices: identifying search frictions and selling pressures," Review of Financial Studies, 25, 1155–1206.

Friewald, N., R. Jankowitsch, and M. G. Subrahmanyam, 2012, "Illiquidity or credit deterioration: A study of liquidity in the US corporate bond market during financial crises," Journal of Financial Economics, 105, 18–36.

Gilson, S. C., K. John, and L. H. P. Lang, 1990, "Troubled debt restructurings*1: An empirical study of private reorganization of firms in default," Journal of Financial Economics, 27, 315–353.

Gomes, J. F., and L. Schmid, 2010, "Equilibrium Credit Spreads and the Macroeconomy," working paper.

He, Z., and A. Krishnamurthy, 2013, "Intermediary asset pricing," American Economic Review, 103, 732–770.

He, Z., and K. Milbradt, 2014, "Endogenous Liquidity and Defaultable Bonds," Forthcoming, Econometrica.

He, Z., and K. Milbradt, 2015, "Dynamic Debt Maturity," Working Paper.

He, Z., and W. Xiong, 2012a, "Dynamic debt runs," Review of Financial Studies, 25, 1799–1843.

He, Z., and W. Xiong, 2012b, "Rollover risk and credit risk," The Journal of Finance, 67, 391–430.

Huang, J.-Z., and M. Huang, 2012, "How much of the Corporate-Treasury Yield Spread is Due to Credit Risk," Review of Asset Pricing Studies, 2, 153–202.

Ivashina, V., D. C. Smith, and B. Iverson, 2013, "The Ownership and Trading of Debt Claims in Chapter 11 Restructurings," Working Paper.

Jankowitsch, R., F. Nagler, and M. G. Subrahmanyam, 2013, "The Determinants of Recovery Rates in the US Corporate Bond Market," Working Paper.

Krishnamurthy, A., 2002, "The bond/old-bond spread," Journal of Financial Economics, 66, 463–506.

Leland, H., 1998, "Agency costs, risk management, and capital structure," Journal of Finance, 53, 1213–1243.

Leland, H., and K. Toft, 1996, "Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads," The Journal of Finance, 51, 987–1019.

Liu, J., F. A. Longstaff, and R. E. Mandell, 2006, "The Market Price of Risk in Interest Rate Swaps: The Roles of Default and Liquidity Risks," The Journal of Business, 79, 2337–2360.

Longstaff, F., S. Mithal, and E. Neis, 2005, "Corporate yield spreads: Default risk or liquidity? New evidence from the credit default swap market," The Journal of Finance, 60, 2213–2253.

Longstaff, F. A., 2004, "The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices," The Journal of Business, 77, 511–526.

Oehmke, M., and A. Zawadowski, 2013, "The anatomy of the CDS market," Working Paper.

Roll, R., 1984, "A simple implicit measure of the effective bid-ask spread in an efficient market," Journal of Finance, 39, 1127–39.

Suarez, G., E. Schroth, and L. Taylor, 2014, "Dynamic Debt Runs and Financial Fragility: Evidence from the 2007 ABCP Crisis," Journal of Financial Economics, 112, 164–189.

Appendix

A Holding Costs Microfoundation

This section gives the details of the derivation of endogenous holding costs that depend on current bond value. For simplicity, we ignore the time-varying aggregate state. Suppose that investors can only borrow at the riskfree rate r if the loan is collateralized; otherwise the borrowing rate is $r + \chi$ for all uncollateralized amounts. Suppose further that, when an investor is hit by a liquidity shock, he needs to raise an amount of cash that is large relative to his financial asset holdings. This implies that the investor will borrow at the uncollateralized rate $r + \chi$ in addition to selling all of his liquid assets.

The investor can reduce the financing cost of uncollateralized borrowing by using the bond as collateral to raise an amount $(1 - h(y)) P(y)$, where $h(y)$ is the haircut on the collateral and $P(y) = \frac{A(y)+B(y)}{2}$ is the midpoint bond price. Then, the ownership of the bond conveys a **marginal** value of $\chi (1 - h(y)) P(y)$ per unit of time (equaling to the net savings on financing cost) until the time of sale. At the time of sale, which occurs with intensity λ, on top of the sale proceeds equal to the bid price $B(y)$, the bond conveys a **marginal** value of $\chi B(y)$ per unit of time perpetually, or $\frac{\chi B(y)}{r}$ in present value. Notice that there is no haircut on the cash proceeds. Intuitively, a more risky collateral asset, due to a greater haircut, lowers its marginal value for an investor hit by liquidity shocks. This is the channel that generates endogenous holding costs in our model.

We now characterize value of the bond in terms of the value (utility) of an investor, which can be different from the market price of the bond when the investor's marginal value of cash is above 1.

$$rV_H(y) = c + \mathcal{L}V_H(y) + \xi_{HL}[V_L(y) - V_H(y)], \quad (18)$$

$$rV_L(y) = c + \chi(1 - h(y))P(y) + \mathcal{L}V_L(y) + \lambda\left[B(y) + \frac{\chi B(y)}{r} - V_L(y)\right], \quad (19)$$

where $\mathcal{L}$ stands for the standard differential operator for the geometric Brownian motion of cashflows. Suppose that with probability β, the investor can make a take-it-or-leave-it offer to the dealer, and with probability $(1-\beta)$ the dealer can make the offer to the investor. If the dealer gets to make the offer, his offering price, denoted by $B_d(y)$, should satisfy $B_d(y) + \frac{\chi B_d(y)}{r} - V_L(y) = 0$, which implies that

$$B_d(y) = \frac{r}{r+\chi}V_L(y). \quad (20)$$

The dealer's outside option is 0, and his valuation of the bond is simply $V_H(y)$, the price at which he can sell the bond on the secondary market to H-type investors. If the investor gets to make the offer, his offering price, denoted by $B_i(y)$, will be

$$B_i(y) = V_H(y). \quad (21)$$

Thus, with probability β, a surplus of $\left[(1+\frac{\chi}{r})V_H(y) - V_L(y)\right]$ accrues to the investor, and with probability $(1-\beta)$, zero surplus accrues to the investor. We can also see that cash has a Lagrange multiplier $1 + \frac{\chi}{r} > 1$ in the liquidity state L.

Further, the mid-point bond price is

$$P(y) = \frac{A(y) + B(y)}{2} = \frac{V_H(y) + (1-\beta)V_H(y) + \beta\frac{r}{r+\chi}V_L(y)}{2} = \left(1 - \frac{\beta}{2}\right)V_H(y) + \frac{\beta}{2}B_d(y). \quad (22)$$

Multiplying the V_L equation (19) by $\frac{r}{r+\chi}$, we rewrite to get

$$rB_d(y) = \frac{r}{r+\chi}[c + \chi(1-h(y))P(y)] + \mathcal{L}B_d(y) + \lambda\beta[V_H(y) - B_d(y)] \quad (23)$$

From (19) to (23), we have simply re-expressed the bond valuation in state L from being in utility terms into dollar terms through the Lagrange multiplier, which allows us to express the effective holding cost in dollars. Specifically, we can rewrite the flow term in (23) as

$$\frac{r}{r+\chi}[c + \chi(1-h(y))P(y)] = c - \underbrace{\frac{\chi}{r+\chi}[c - r(1-h(y))P(y)]}_{\text{holding cost}},$$

where the second term can be interpreted as the holding cost. Under appropriate parameterization, this holding cost is increasing in the spread for uncollateralized financing χ and the haircut $h(y)$.

While we have left the haircut function $h(y)$ as exogenous, it is intuitive that it should become larger when the bond becomes more risky, which is when the bond price is lower. Consider the following functional form,

$$h\left(y\right)=\frac{a_{0}}{P\left(y\right)}-a_{1}.$$

By choosing $a_0 = (N(r+\chi) - c)/r$ and $a_1 = \chi/r$, we obtain the holding cost $hc\left(y\right) = \chi\left(N - P\left(y\right)\right)$ as in equation (5).

B Model Implied Credit Default Swap

Since the CDS market is much more liquid than that of corporate bonds, following Longstaff, Mithal, and Neis (2005) we compute the model implied CDS spread under the assumption that the CDS market is perfectly liquid.[35] Let τ (in years from today) be the time of default. Formally, if today is time u, then $\tau \equiv \inf\{t : y_{u+t} \leq y_{def}^{s_{u+t}}\}$ can be either the first time at which the log cash-flow rate y reaches the default boundary y_{def}^{s} in state s, or when $y_{def}^{G} < y_t < y_{def}^{B}$ so that a change of state from G to B triggers default. Thus, for a T-year CDS contract, the required flow payment f is the solution to the following equation:

$$\mathbb{E}^{\mathcal{Q}}\left[\int_{0}^{\min[\tau,T]}\exp\left(-rt\right)f dt\right]=\mathbb{E}^{\mathcal{Q}}\left[\exp\left(-r\tau 1_{\{\tau\leq T\}}\right)LGD_{\tau}\right], \tag{24}$$

where LGD_τ is the loss-given-default, which is the bond face value p minus its recovery value, where the recovery value is defined as the mid transaction price at default. If there is no default, no loss-given-default is paid out by the CDS seller. We calculate the flow payment f that solves (24) using a simulation method. The CDS spread, f/p, is defined as the ratio between the flow payment f and the bond's face value p.

C The Case of More Frequent Liquidity Shocks

[35] Arguably, the presence of the CDS market will in general affect the liquidity of the corporate bond market; but we do not consider this effect. A recent theoretical investigation by Oehmke and Zawadowski (2013) shows ambiguous results in this regard. Further, there is some ambiguity in the data about which way the illiquidity in the CDS market affects the CDS spread. Bongaerts, De Jong, and Driessen (2011) show that the sellers of CDS contracts earn a liquidity premium.

Figure 6: **Time-Series Structural Decomposition of Credit Spreads for Baa and Ba-rated Firms with More Frequent Liquidity Shocks.** Liquidity shock intensities are $\xi_G = 1.4$ and $\xi_B = 2$ which double the benchmark liquidity shock intensities in Table 1. We also adjust the holding cost intercept down from $N = 115$ to $N = 110$ to deliver similar total credit spreads for Baa ratings.

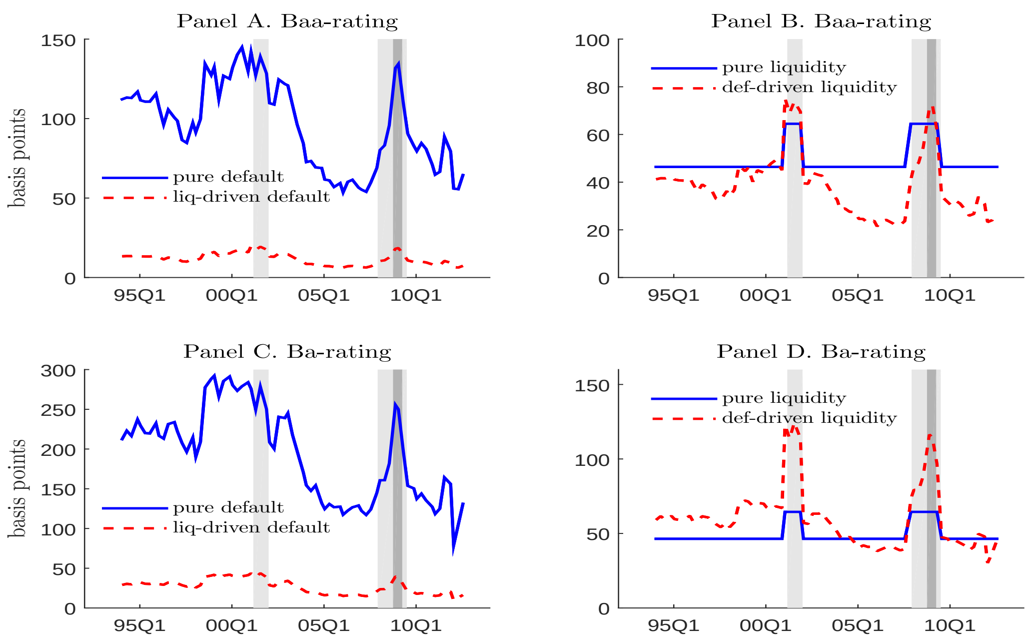


Table 1: **Baseline parameters used in calibration.** Unreported parameters are the tax rate of $\pi = 0.35$, and bond face value $p = 100$. Panel A reports pre-fixed parameters. We explain how we pick these parameter values in Section 4.1. Panel B reports four calibrated parameters. The idiosyncratic volatility σ_f, the holding cost intercept N, and holding cost slopes χ_s are set to target Baa default probability, investment grade bid-ask spreads in both states, and superior grade bid-ask spread in state G.

Symbol	Description	State G	State B	Justification / Target
		A. Pre-fixed parameters		
$\zeta^{\mathbb{P}}$	Transition density	0.1	0.5	literature
$\exp(\kappa)$	Jump risk premium	2.0	0.5	literature
η	Risk price	0.17	0.22	literature
r	Risk free rate	0.05		nominal riskfree rate
$\mu_{\mathbb{P}}$	Cash flow growth	0.045	0.015	literature
σ_m	Systematic vol	0.10	0.11	equity volatility
ω	Primary market issuance cost	0.01		literature
m	Average maturity intensity	0.2		literature
Δ	Treasury liquidity premium	15 bps	40 bps	repo-Treasury spread
λ	Meeting intensity	50	20	anecdotal evidence
ξ	Liquidity shock intensity	0.7	1.0	bond turnover rate
β	Investor's bargaining power	0.05		literature
α_H	Recovery rate of H type	58.71%	32.56%	bid-ask for defaulted bonds
α_L	Recovery rate of L type	57.49%	30.50%	bid-ask for defaulted bonds
		B. Calibrated parameters		
σ_f	Idiosyncratic vol	0.25		Baa default probability
N	Holding cost intercept	115		Investment bid-ask spread
χ	Holding cost slope	0.06	0.11	Superior bid-ask spread in G

Table 2: **Default probabilities and credit spreads across credit ratings.** Default probabilities are cumulative default probabilities over 1920-2011 from Moody's investors service (2012), and credit spreads are from FISD and TRACE transaction data over 1994-2010. We report the time series mean, with the standard deviation (reported underneath) being calculated using Newey-West procedure with 15 lags. The standard deviation of default probabilities are calculated based on the sample post 1970's due to data availability issue. When calculating the model-implied moments, we pick the cash-flow y to exactly match the cross-sectional distribution of model-implied market leverage with the empirical counterpart for Compustat firms (excluding financial and utility firms) for each rating category in each quarter over 1994-2010 (excluding the crisis quarters 2008Q4 and 2009Q1).

	Maturity = 5 years				Maturity = 10 years			
	Aaa/Aa	A	Baa	Ba	Aaa/Aa	A	Baa	Ba
	Panel A. Default probability (%)							
data	0.7	1.3	3.1	9.8	2.1	3.4	7.0	19.0
model	0.3	0.8	2.4	7.4	1.6	3.9	7.9	15.9
	Panel B. Credit spreads (bps)							
	State G							
data	55.7	85.7	149	315	61.2	90.2	150	303
	(3.7)	(6.6)	(15.5)	(33.8)	(4.4)	(6.3)	(12.8)	(22.7)
model	57.7	73.4	114	237	86.0	122	182	301
	State B							
data	107	171	275	542	106	159	262	454
	(5.8)	(10.5)	(23.9)	(29.8)	(6.7)	(13.8)	(29.3)	(44.4)
model	112	135	191	343	136	185	261	404

Table 3: **Comparative statics and comparison to alternative models.** We focus on the calibration results for 10-year credit spreads, and the rows of "baseline"are taken from Table 2 for 10-year bonds. The rows of "$\xi_G = 1$"report the implied moments when we increase the baseline parameter state-G liquidity shock intensity from $\xi_G = 0.7$ to $\xi_G = \xi_B = 1$. The rows of "$\chi_s = 0$ "report the implied moments when we eliminate the secondary market illiquidity by setting $\chi_s = 0$ for $s = G, B$. The rows of "hc_s "are under the assumption that holding costs depend on aggregate state only (i.e., not $hc_s(P)$, and we calibrate $hc_G = 1.38$ and $hc_B = 2.32$ to match investment-grade bid-ask spreads). When calculating the model-implied moments, we pick the cashflow y to exactly match the cross-sectional distribution of model-implied market leverage with the empirical counterpart for Compustat firms (excluding financial and utility firms) for each rating category in each quarter over 1994-2010 (excluding the crisis quarters 2008Q4 and 2009Q1).

	Aaa/Aa	A	Baa	Ba
	Panel A. Default probability (%)			
baseline	1.59	3.88	7.90	15.88
$\xi_G = 1$	1.62	3.92	8.01	16.17
$\chi_s = 0$	1.40	3.54	7.40	15.40
hc_s	1.51	3.66	7.52	15.45
	Panel B. Credit spreads (bps)			
	State G			
baseline	86.0	122	182	301
$\xi_G = 1$	92.7	130	192	315
$\chi_s = 0$	32.5	57.5	103	200
hc_s	95.3	126	176	278
	State B			
baseline	136	185	261	404
$\xi_G = 1$	141	191	268	414
$\chi_s = 0$	59.5	90.7	143	248
hc_s	146	185	245	359

Table 4: **Bid-ask spreads across credit ratings.** The normal time bid-ask spreads are taken from Edwards, Harris, and Piwowar (2007) for median sized trades. The numbers in recession are normal time numbers multiplied by the empirical ratio of bid-ask spread implied by Roll's measure of illiquidity (following Bao, Pan, and Wang (2011)) in recession time to normal time. The row of "baseline "is the model implied bid-ask spread, computed for a bond with time to maturity of 8 years, which is the mean time-to-maturity of frequently traded bonds (where we can compute a Roll (1984) measure) in the TRACE sample. The rows of "hc_s "are under the assumption that holding costs depend on aggregate state only (i.e., not $hc_s(P)$, and we calibrate $hc_G = 1.38$ and $hc_B = 2.32$ to match investment-grade bid-ask spreads). When calculating the model-implied moments, we pick the cashflow y to exactly match the cross-sectional distribution of model-implied market leverage with the empirical counterpart for Compustat firms (excluding financial and utility firms) for each rating category in each quarter over 1994-2010 (excluding the crisis quarters 2008Q4 and 2009Q1).

	State G			State B		
	Superior	Investment	Junk	Superior	Investment	Junk
data	40	50	70	77	125	218
baseline	39	50	61	111	125	186
hc_s	45	50	49	123	125	138

Table 5: **Structural Liquidity-Default Decomposition for 10-Year Bonds Across Ratings.** We perform the structural liquidity-default decomposition for a 10-year bond following Section 4.3.1, given rating and aggregate state, and then aggregate over the empirical leverage distribution in Compustat. The reported credit spreads are relative to the risk-free rate (indicated by rf). We also report the model implied CDS spreads across ratings.

Rating	State	Credit Spread (rf)	Structural Decomposition				CDS Spread
			Pure Def	*Liq* $\rightarrow$ *Def*	*Pure Liq*	*Def* $\rightarrow$ *Liq*	
Aaa/Aa	*G* (bps)	71	20	2	45	5	23
	(%)		27	3	63	7	33
	B (bps)	96	22	2	63	9	27
	(%)		23	2	66	9	28
A	*G* (bps)	107	46	4	45	12	54
	(%)		43	4	42	11	50
	B (bps)	145	55	4	63	22	66
	(%)		38	3	44	15	46
Baa	*G* (bps)	167	93	7	45	22	106
	(%)		56	4	27	13	64
	B (bps)	221	109	9	63	41	129
	(%)		49	4	29	18	58
Ba	*G* (bps)	286	192	13	45	38	225
	(%)		67	4	16	13	79
	B (bps)	364	215	16	63	70	257
	(%)		59	5	17	19	71

Table 6: **Effect of Liquidity Provision Policy on 10-Year Bonds Across Ratings.** We consider a policy experiment that improves the liquidity environment (χ and λ) in the B state to be as good as G state (i.e., $\chi_B = 0.06$ and $\lambda_B = 50$). We fix the distribution of cash flow levels y at the values that deliver the observed market leverage distribution in Compustat (excluding financial and utility firms) for the corresponding state in our baseline calibration. We then report the average credit spreads (relative to the risk-free rate) under the policy for each state together with credit spread without policy. We perform the structural liquidity-default decomposition to examine the channels that are responsible for the reduced borrowing cost. We report the percentage contribution of each component to the credit spread change.

Rating	State	Credit Spread (rf)		Contribution of Each Component		
		w/o. policy	w. policy	pure LIQ (%)	LIQ→DEF (%)	DEF→LIQ (%)
Aaa/Aa	G	71.1	41.9	83	5	12
	B	96.0	44.4	83	3	14
A	G	107	71.9	69	8	23
	B	145	82.1	68	5	27
Baa	G	167	125	57	10	33
	B	221	143	55	7	38
Ba	G	286	234	46	12	42
	B	364	262	42	9	49