

Venture Capital and the Macroeconomy*

Christian C. G. Opp[†]

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Abstract

I develop a model of venture capital (VC) intermediation that can explain central empirical facts about the magnitude and cyclical nature of VC activity, and allows evaluating its impact on the macroeconomy. The framework reveals how pro-cyclical VC investment dynamics are self-reinforcing through a risk premium channel that rationalizes strongly declining funding standards in booms. VC investments' growth contributions generate significant societal value added, despite their strong cyclical nature, even after accounting for large uninsurable idiosyncratic risks associated with VC contracts. The proposed general equilibrium model yields exact solutions despite the presence of informational frictions, imperfect risk sharing, and endogenous growth.

Keywords: Venture Capital, Growth, Innovation, Intermediation, Boom-Bust Cycles, Risk Premia, Idiosyncratic Risk

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[†]The Wharton School, University of Pennsylvania. Email: opp@upenn.edu.

1. Introduction

Innovation by new firms has long been viewed as central to economic growth and development (Schumpeter, 1934). Over the past few decades venture capital funds have emerged as prominent financial intermediaries facilitating the entry of young innovative firms. According to Kaplan and Lerner (2010), over 60 percent of initial public offerings (IPOs) have been VC-backed between 1999 and 2009, and four of the twenty companies with the highest market capitalization in the United States have been funded by venture capital. Furthermore, empirical studies find that venture capital has had significant influence on patented inventions.¹ Despite this remarkable record, the VC industry also has gone through dramatic boom-bust cycles that appear anomalous to many observers, casting doubts on the positive macroeconomic effects of VC intermediation. Gupta (2000) and Gompers and Lerner (2003) argue for example that extreme movements in VC industry activity are a symptom of irrational overreaction, resulting in periods in which too many firms are funded, followed by ones in which not enough firms have access to capital.² In addition, a growing literature has highlighted potential reasons why financial sector employment and compensation might be excessive,³ a debate particularly relevant for the VC industry, which has historically generated large fees.⁴

Despite the potential macroeconomic relevance of venture capital the literature lacks to date a quantitative framework that can explain central empirical observations relating to the magnitude and cyclicity of venture capital activity, and allows evaluating its contribution to the macroeconomy. In this paper I aim to take a step toward developing a unified macro-finance framework that fills this void. The model features three building blocks that are essential to achieving this goal: (1) A macroeconomic environment where innovations by new firms can generate endogenous economic growth. (2) A micro-founded representation of the VC industry that captures central features of the industry's market structure and economic function, allowing the model to match empirical measures of VC intermediation such as VC commitments, VC manager compensation, and payoffs to entrepreneurs. (3) An environment with empirically plausible asset pricing implications that captures agents' revealed attitudes toward growth and risk, and allows appropriately interpreting information

¹See, e.g., Kortum and Lerner (2000) and Samila and Sorenson (2011) for empirical evidence.

²See also Gompers and Lerner (2000, 2001, 2003) for evidence on lenient funding standards, high valuations, and high capital commitments during venture capital booms. Korteweg and Sorensen (2010) and Kaplan and Lerner (2010) also highlight the negative relationship between capital flows to VC funds and future returns.

³See, e.g., Philippon (2010), Glode, Green, and Lowery (2012), Philippon and Reshef (2012), and Philippon (2015) for papers analyzing financial sector compensation.

⁴See, e.g., Hall and Woodward (2007), Kaplan and Rauh (2010), and Greenwood and Scharfstein (2013).

encoded in market prices.

The proposed framework can draw on a variety of empirical data on VC intermediation, asset prices, and macroeconomic dynamics to inform the model’s parameters. The calibrated model successfully matches central empirical facts about the magnitude and strong cyclicity of aggregate VC fund commitments, VC-backed IPOs and mergers and acquisitions (M&As), VC fund fee revenues, and payoffs to entrepreneurs, suggesting that these data are not necessarily evidence of investor irrationality. Accounting for the pricing of macroeconomic risks reveals that pro-cyclical entry by VC-funded firms endogenously reinforces itself, as it causes counter-cyclical post-IPO risk premia. The correspondingly lenient funding standards in booms help the model match extreme increases in VC investment, as observed during the internet boom of the late 1990s, as well as the negative empirical relation between VC investment and future returns.

Within this quantitative framework I perform a Lucas (1987) type calculation to evaluate the macroeconomic benefits of VC activity. I find that agents would be willing to give up between 1 and 2 percent of lifetime consumption to avoid losing the growth created by VC investments, significantly exceeding actual commitments to VC funds. Despite the strong cyclicity of VC activity, the analysis thus indicates a potent positive societal impact. Relative to estimates of the cost of business cycles the economic magnitudes are significant (see, e.g., Lucas, 2003, Alvarez and Jermann, 2004) and highlight the relevance of policies affecting the VC industry, which was able to attract material capital commitments only after regulatory reforms and tax changes were implemented in the late 1970s and early 1980s.⁵

I find that, due to imperfect risk sharing, idiosyncratic risk greatly affects the ex ante value entrepreneurs and VC managers assign to performance-sensitive payoffs from VC contracts. The extreme risks typical for venture capital imply that risk averse agents severely discount these risky contractual payoffs, consistent with the findings by Hall and Woodward (2007, 2010). Despite this friction, I find that agents assign significant value to *aggregate* VC activity, in particular due to its positive impact on technological progress, and since diversified investors hold significant fractions of ventures’ financial claims.

The paper proposes a highly tractable general equilibrium environment with informational frictions, imperfect risk sharing, and endogenous growth that could provide a useful stepping stone for future studies demanding such features. The methodology provides exact

⁵According to Kroszner and Strahan (2014), a particularly important regulatory change occurred in 1979, when the “prudent expert” standard was broadened to allow pension funds to invest in VC funds. In addition, in 1981, the Economic Recovery Tax Act lowered the top capital gains tax rate from 28 percent to 20 percent.

solutions, without requiring computationally-intensive methods. In addition, as the model captures both risk-based and non risk-based factors affecting average returns to VC investments, it provides a cautionary note to empirical studies estimating risk-based asset pricing models, adding to other well-known challenges with VC data.

In the model, entrepreneurs generate venture ideas with heterogenous success prospects. VC managers can provide VC funds with industry expertise that allows identifying and supporting the most promising entrepreneurs.⁶ Asymmetric information and heterogenous access to high-quality entrepreneurs imply an imperfectly competitive market structure where VC funds can generate economic profits (“alpha”) that VC managers extract via fees. However, depending on venture productivity and macroeconomic conditions, rents from active VC intermediation can be insufficient to cover the required compensation for fund managers, causing funding fragility and potential barriers to entry for new ventures in downturns.

Yet such barriers to entry also imply that persistent negative shocks to creative destruction and growth can be partially good news for ventures that obtained funding during a preceding boom and successfully entered the market. Although these incumbents’ dividend growth suffers when aggregate growth decreases, they benefit to a first-order degree from persistent declines in the funding and entry of new competitors. Venture investments thus can become hedges against macroeconomic low-frequency risks associated with persistent slow-downs in technological progress, especially in those industries where entry is highly pro-cyclical.⁷ Low discount rates and associated lenient funding standards in booms in turn endogenously reinforce the cyclicity of VC investment and entry.

Related literature. To my best knowledge this is the first paper to develop a quantitative dynamic general equilibrium model that explains central empirical facts about the magnitude and cyclicity of venture capital activity and allows evaluating its impact on the macroeconomy. There is a large literature in macroeconomics and economic growth that addresses technological change but abstracts from venture capital intermediation and the pricing of

⁶Hochberg, Ljungqvist, and Lu (2007) document the importance of VC managers’ network connections for investment opportunity sets and access to information in venture capital markets. Kaplan and Strömberg (2001, 2004) provide direct empirical evidence on VCs’ investment analyses, documenting the importance of VCs’ efforts to evaluate and screen entrepreneurs. Further, Gompers, Kovner, Lerner, and Scharfstein’s (2008) empirical findings highlight the importance of “industry-specific human capital,” including a “network of industry contacts to identify good investment opportunities as well as the know-how to manage and add value to these investments.”

⁷VC activity is strongly correlated with valuations and IPO activity in public equity markets. See, e.g., Gompers and Lerner (1998), Kaplan and Schoar (2005), and Gompers, Kovner, Lerner, and Scharfstein (2008) for empirical evidence. In addition, see Greenwood and Jovanovic (1999), Jovanovic and Rousseau (2003), and Jovanovic and Rousseau (2005) for empirical evidence on low-frequency variation in entry rates.

risk.⁸ The most related papers in this literature are Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (2001), which link expectations about future increases in the rate of creative destruction during technological revolutions to initial stock market declines.⁹ In my model persistent variation in creative destruction also generates the corresponding reverse effect that expectations about future declines in entry amplify preceding booms. In particular, my model reveals that pro-cyclical variation in creative destruction materially reduces venture investments' risk premia, which is a result that would not be captured by a model that abstracts from the pricing of risk. For similar reasons, my paper deviates from the existing literature on finance and growth,¹⁰ such as King and Levine (1993b) who consider project selection in a quality-ladder model with constant aggregate growth. King and Levine's model does not aim to provide a quantitative analysis of VC activity, and also cannot qualitatively speak to objects of interest in this paper, such as cyclical variation in intermediation and funding standards, the impact of aggregate risk and risk pricing, and the performance-sensitive compensation of entrepreneurs and VC managers.

With regards to its implications for risk premia my paper is more closely related to the strand of literature that analyzes general equilibrium production-based models to examine the time-series and cross-sectional properties of returns (see, e.g., Gomes, Kogan, and Zhang, 2003). Several papers in this literature address the asset pricing implications of technological innovation, although none of the existing papers in this literature consider venture capital intermediation and its effects on returns to investors and macroeconomic growth. In Garleanu, Kogan, and Panageas (2012) and Kogan, Papanikolaou, and Stoffman (2015) displacement risk arises as a priced risk factor in economies with imperfect risk sharing and can help explain empirical patterns in asset returns, such as the value premium. While I also analyze the effects of imperfect risk sharing, displacement risk in my setting affects discount rates primarily through its impact on firms' exposures to low-frequency risk associated with persistent changes in aggregate growth. Kung and Schmid (2015) consider an expanding varieties model similar to Comin and Gertler (2006) to analyze the asset pricing implications of persistent movements in productivity in the presence of recursive preferences. The authors provide empirical evidence for the existence of innovation-driven low-frequency movements in aggregate growth rates. Several recent papers further analyze the implications of technology shocks that are embodied in new capital.¹¹ Although the technology considered in my paper differs from the ones in these papers, similarities arise, as growth

⁸See, e.g., Jovanovic and MacDonald (1994), Laitner and Stolyarov (2003, 2004).

⁹A similar initial stock market drop is obtained in the model of Laitner and Stolyarov (2003).

¹⁰See, e.g., Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), King and Levine (1993a), Levine and Zervos (1998), Rajan and Zingales (1998), and Townsend and Ueda (2006).

¹¹See Papanikolaou (2011), Garleanu, Panageas, and Yu (2012), Kogan and Papanikolaou (2013, 2014).

is affected by shocks impacting the development of new, higher-quality goods that displace older vintages of goods. Finally, Pastor and Veronesi (2009) highlight that the endogenous adoption of a revolutionary technology is initially associated with low discount rates and increasing expected cash flows, but over time, risk premia rise and eventually lead to falling asset prices as the nature of risk changes from idiosyncratic to systematic. In my model, declining discount rates in booms increase valuations and VC investment, consistent with empirical evidence that funding booms predict low returns (see footnote 2).

As external financing is a central element of my model, my paper is also related to the strand of literature that analyzes asset pricing implications of costly external finance (see, e.g., Gomes, Yaron, and Zhang, 2003). However, in contrast to this literature, my model focuses on the financing of new ventures and explicitly models informational frictions that VC funds address. Further, my paper speaks to the general equilibrium dynamics of IPO volume and is thus related to theories of IPO waves, such as Jovanovic and Rousseau (2001) and Pastor and Veronesi (2005).

Although the framework proposed in this paper captures a multitude of economic forces related to venture capital intermediation and its linkages to the macroeconomy, it naturally also abstracts from a variety of economic forces that can create cyclicity, such as entrepreneurs' self-fulfilling expectations about the implementation of innovations (Shleifer, 1986), rational herd behavior (Scharfstein and Stein, 1990), endogenous timing and learning (Gul and Lundholm, 1995), firm-specific learning-by-doing (Stein, 1997), high investment in new technologies due to incentives to learn about the curvature of the production function (Johnson, 2007), low expected returns and high investment in risky technologies in the presence of relative wealth concerns (DeMarzo, Kaniel, and Kremer, 2007), and complementarities in investors' information production (Dow, Goldstein, and Guembel, 2011). Finally, my paper abstracts from behavioral frictions, which might also contribute to the cyclical behavior of the VC industry.

2. The Economy

2.1. Agents

The economy is populated by infinitely-lived heterogenous agents with distinct skills, referred to as *entrepreneurs*, *network VCs*, *white-collar labor*, and *blue-collar labor*. Subscripts $S \in \{E, V, W, B\}$ index these four skill types. Entrepreneurs generate venture ideas. Network VCs can help identify the most promising entrepreneurs. White-collar labor can be employed by firms to implement venture ideas, or by VC funds to facilitate intermediation. Blue-collar labor is required to produce intermediate goods based on existing patents. There is a double-continuum of agents with each skill type S . Each agent is identified by a tuple (S, i_G, i_S) that specifies the skill type, S , a group index, $i_G \in [0, 1]$, and a skill subgroup index, $i_S \in [0, 1]$. The set of all agents in the economy is given by the Cartesian product $\Phi = \{E, V, W, B\} \times [0, 1] \times [0, 1]$. At date 0, agents own identical shares of existing firms and are endowed with their skill.

2.2. Preferences

The presented setup can be thought of as the continuous-time counterpart of a discrete-time economy, where a period lasts for an interval of Δt , and where I consider the limiting case $\Delta t \rightarrow 0$. In this limit, agents' preferences are described by stochastic differential utility (Duffie and Epstein, 1992a,b), which is a continuous-time version of the recursive preferences of Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). These preferences allow specifying risk aversion and intertemporal elasticity of substitution separately, which is key to matching asset pricing moments that are important for the analysis. The utility index over a consumption process C is defined as:

$$J_t = \mathbb{E}_t \left[\int_t^\infty m(C_\tau, J_\tau) d\tau \right]. \quad (1)$$

Here the function $m(C, J)$ is a normalized aggregator of current consumption and continuation utility that takes the form

$$m(C, J) = \frac{\beta}{\rho} \left(((1 - \gamma)J)^{1 - \frac{\rho}{1 - \gamma}} C^\rho - (1 - \gamma)J \right), \quad (2)$$

with $\rho = 1 - \frac{1}{\psi}$, where $\beta > 0$ is the rate of time preference, $\gamma > 0$ is the coefficient of relative risk aversion, and $\psi > 0$ is the elasticity of intertemporal substitution.

2.3. Labor Markets

Blue-collar labor and white-collar labor are supplied inelastically. Each agent with these skills is endowed with one unit of labor, such that the aggregate supply of each type of labor is one. Blue-collar and white-collar labor obtain distinct, potentially time-varying, equilibrium wage rates, denoted by w_B and w_W respectively. The market for entrepreneurs and network VCs will be described in Sections 2.4 and 2.5.

2.4. Production and Development of Intermediate Goods

The production of a final consumption good uses a continuum of intermediate goods. In this section, I discuss the production of intermediate goods based on existing patents and the process by which ventures can innovate provided they obtain funding. In Section 2.5, I describe how VC funds operate and how ventures can obtain funding. In Section 2.6, I describe final good production.

Intermediate good production. Intermediate good varieties, also referred to as industries or product lines, are indexed by $v \in \Psi = [0, 1]$. A firm that owns a patent for an intermediate good needs to employ one unit of blue-collar labor to manufacture one unit of the intermediate good.

Quality ladder. The description of innovative activity in the model builds on the quality-ladder technology of Schumpeterian growth models (Grossman and Helpman, 1991). This environment is a fitting starting point, as it captures the basic empirical regularity that innovative activity by new firms leads to creative destruction and endogenous economic growth.¹² Process innovations can lead to quality improvements for intermediate goods. Let $q(v, t)$ denote the quality of the best intermediate good available in industry v at time t , and let $M(v, t)$ be a counting process that denotes the number of innovations that occurred in industry v between time 0 and time t . A quality ladder determines the evolution of the

¹²See Greenwood and Jovanovic (1999), Jovanovic and Rousseau (2003), and Jovanovic and Rousseau (2005) for empirical evidence on the importance of entry by young firms and creative destruction in times of rapid technological change, such as the Electrification and IT eras.

best available intermediate good in each industry:

$$q(v, t) = \kappa^{M(v,t)} q(v, 0), \quad \forall v \text{ and } t, \quad (3)$$

where $\kappa > 1$ and $q(v, 0) \in \mathbb{R}_+$. The quality ladder implies that each innovation leads to a proportional quality increase by a factor κ . If a firm innovates it obtains a perpetual patent for the new intermediate good. Yet, as standard in quality-ladder models, the patent system allows other firms to use this know-how to develop new intermediate goods of even higher quality. Further, the economy features the standard restriction that firms owning patents of different qualities cannot contract to share the higher monopoly profits that could be earned through collusion.¹³

Inputs required for innovative activity. Two inputs are required to undertake a venture aimed at developing a new patent: (1) a venture idea generated by an entrepreneur or an existing firm, and (2) one unit of white-collar labor that implements this venture idea. Entrepreneurs and existing firms can generate any number of venture ideas but are heterogeneous in their abilities to generate high-quality ideas. In particular, entrepreneurs can obtain temporary productivity shocks allowing them to generate venture ideas in a specific industry that have better prospects of yielding innovations than those generated by all other entrepreneurs or firms. However, these *most productive entrepreneurs* have to be identified, funded, and supported by a VC fund to realize their superior opportunities, a process described below in Section 2.5.

Which entrepreneur can generate better ideas in a given industry varies across periods and is determined by the following matching relation: the most productive entrepreneur in industry v in period $[t, t + dt]$ has the entrepreneur index $i_E = u_{E,t}$, with $u_{E,t} \sim \text{unif}(0, 1)$, and the group index

$$i_G = u_{E,t} + v - \mathbf{1}_{\{v+u_{E,t}>1\}}, \quad (4)$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. This particular matching technology makes the setup particularly tractable when risk sharing is limited, which is considered in Section 5.

Innovation at the industry level. Before specifying the success prospects of individual venture ideas it is useful to define the arrival rate of innovations at the *industry level*. Suppose

¹³This restriction is also customary in the patent-race literature (see, e.g., Tirole, 1988, Reinganum, 1989).

that a total measure n of efficiency-adjusted venture ideas are implemented in a given period, where n will be defined explicitly below. The Poisson arrival rate of a new innovation in this industry is given by:

$$h(v, Z, n) = \theta(v, Z) n^\eta, \quad (5)$$

where $0 < \eta < 1$, and where $\theta(v, Z)$ is an exogenous industry-specific productivity process that is governed by the aggregate state variable Z . The state Z follows a time-homogeneous continuous time Markov chain taking values in the set $\Omega = \{1, \dots, l\}$. The transition rates between states are denoted by $\lambda_{ZZ'}$ and are collected in the generator matrix Λ . I also define $l \cdot (l-1)$ counting processes $N_t(Z, Z')$ that keep track of the number of Markov chain switches between all states that have occurred between time 0 and time t .

Persistent stochastic variation in productivity across industries is essential to the paper's objective to analyze cyclical behavior. The functional specification for h features diminishing returns to scale at the industry level, reflecting the notion that ventures undertaken in the same industry are effectively competing for similar innovations. The decreasing returns to scale parameter η will crucially affect the rents VCs obtain from intermediating funds to the most productive entrepreneurs.

Venture success prospects. Suppose that a measure n_1 of the most productive entrepreneur's venture ideas is undertaken. The Poisson intensity with which one of these ideas succeeds is given by:

$$h_1(v, Z, n_1) = \theta(v, Z) n_1^\eta. \quad (6)$$

In contrast, if a measure n_2 of regular venture ideas — generated by any other entrepreneur or firm — is implemented, one of these ventures succeeds with Poisson intensity:

$$h_2(v, Z, n_1, n_2) = h(v, Z, n_1 + \phi n_2) - h_1(v, Z, n_1), \quad (7)$$

where $\phi \in (0, 1)$ represents an efficiency discount. In case a Poisson arrival corresponding to (7) occurs, ventures in the measure n_2 have a uniform conditional probability of being the winner of the new patent. I define the total efficiency-adjusted measure of ventures as $n \equiv n_1 + \phi n_2$.

The venture success prospects defined in equations (6) and (7) differ in two ways. First, whereas the success prospects of the most productive entrepreneur, h_1 , depend only on the quantity of his own ideas that are implemented, n_1 , regular ideas' success prospects, h_2 , depend on both n_1 and n_2 . This difference can be interpreted as an intra-period first-mover

advantage for the most productive entrepreneur: due to decreasing marginal returns at the industry-level the most productive entrepreneur can “skim off” venture ideas with the highest marginal success rates, leaving the remaining less productive ideas to regular entrepreneurs and firms. The second difference is the efficiency discount ϕ applied to regular projects. This discount implies that a given measure of regular ventures is always (independent of n_1) less productive than an identical measure of ideas generated by the most productive entrepreneur. This feature will allow the model to capture empirical evidence that VC firms screen ventures and improve the efficiency of capital allocation (Kortum and Lerner, 2000). Whereas the first difference ensures that the most productive entrepreneur always generates better ideas — but has a zero sum effect on the industry arrival rate h holding n fixed — the second difference implies a socially beneficial allocative effect from intermediating funds to the most productive entrepreneurs. Overall, the two differences thus affect both the private and social value of VC intermediation.

2.5. Venture Funding

Consistent with the findings of the empirical literature, VC funds in the model can obtain access to “proprietary deal flow” by hiring VC managers with network connections and industry expertise that can help identify the most promising entrepreneurs.¹⁴

VC funds. VC funds are specialized financial institutions that raise funds from investors to finance ventures of a particular industry and vintage, where a vintage refers to the set of ventures of a specific period. Rational fund investors demand an after-fee risk-return profile that is at least as good as the one attainable in competitive financial markets. Ventures that succeed in developing a new patent are sold by a VC fund through an IPO or an M&A transaction.¹⁵ The proceeds from these transactions are paid out to fund share holders. Each VC fund maximizes the market value of expected proceeds from IPOs and acquisitions, net of fund managers’ and entrepreneurs’ compensation and venture investment costs. There is free entry for new VC funds in each period.

VC fund industry expertise. VC funds have to acquire industry expertise to be able to identify and support the most productive entrepreneur in a given industry and vintage.

¹⁴See footnote 6 for papers documenting these characteristic features of the VC industry’s market structure.

¹⁵Ventures and their new patent may be acquired by other existing firms, provided that each of these firms has only a finite number of patents corresponding to a zero measure of industries.

To acquire industry expertise a fund must hire two types of human capital: (1) a network VC that has connections in the industry, and (2) agents with c_i units of white-collar labor that work as fund managers supporting the selection and funding process. I assume that c_i can vary across industries v . A network VC with industry connections has to identify and contact the most productive entrepreneur in the industry in order to negotiate a funding contract and support the implementation of venture ideas. A network VC can provide this service only in one industry per period and can work for only one VC fund at a time.

As agents require compensation commensurate with their outside options in the labor market it is possible that no VC fund optimally hires the talent needed to acquire industry expertise in a particular industry and period. Let $\iota_1(v, t) \in \{0, 1\}$ denote an indicator variable that takes the value 1 if, in equilibrium, a VC fund acquires expertise for the date t vintage in industry v . The most productive entrepreneur's venture ideas cannot be implemented if there is no VC fund with the expertise to identify and support the entrepreneur.

Network VCs and connections. Network VCs can obtain temporary positive shocks that render their connections useful in certain periods. The matching process that determines which network VCs have useful connections is similar to the one for entrepreneurs. The network VC that has connections in industry v and period $[t, t + dt]$ has the network VC index $i_V = u_{V,t}$, with $u_{V,t} \sim \text{unif}(0, 1)$, and the group index

$$i_G = u_{V,t} + v - \mathbf{1}_{\{v+u_{V,t}>1\}}. \quad (8)$$

This matching relation implies that, with positive probability, a network VC receives useful connections repeatedly over time. In addition, in every industry v , there is always one network VC that has these useful connections, although the identities of these network VCs vary over time. As described above, VC funds may optimally refrain from acquiring industry expertise such that a network VC's connections may stay unused in equilibrium (when $\iota_1(v, t) = 0$). Further, just like the matching relation for entrepreneurs, this specification ensures a tractable analysis of the model once risk sharing is imperfect (see Section 5).

Bargaining over VC surplus. Competition among VC funds for talent implies that the compensation for connected network VCs and the most productive entrepreneurs is bid up until these agents can extract the private surplus created from funding the most productive entrepreneur. The allocation of this surplus, if available, is determined by Nash (1950) bargaining between the connected network VC and the most productive entrepreneur in a

given industry. A network VCs' bargaining power is given by $\varrho \in [0, 1]$. Network VCs and entrepreneurs are compensated with venture equity claims.¹⁶

Market structure. All entities that fund *regular* venture ideas (e.g., existing firms) have identical investment opportunities, act competitively, and fund venture ideas as long as they can make at least zero profits. Thus, these entities do not internalize external diminishing returns of regular venture ideas specified in equation (7). Yet, consistent with the interpretation of (7) as an intra-period first-mover advantage for the most productive entrepreneur, entities funding regular venture ideas can condition their decisions on the period's realization of n_1 .¹⁷ In contrast, a VC fund with industry expertise obtains an exclusive relationship with the most productive entrepreneur in an industry, and internalizes diminishing returns to scale among that entrepreneur's ideas (see the definition of h_1 in equation (6)). Ceteris paribus, this effect reduces the quantity of ventures a VC firm with expertise funds. On the other hand, a most productive entrepreneur's venture ideas are more efficient. To streamline the analysis, I will maintain the assumption that $\phi < \eta$, which ensures that the efficiency channel dominates: holding market prices of patents fixed, a VC fund with industry expertise funds strictly more efficiency-adjusted venture ideas than competitive investors that fund regular venture ideas.

2.6. Final Good Production

The final good is produced by competitive firms according to the Cobb-Douglas production function

$$Y_t = A_t \cdot \exp \left(\int_{\Psi} \log [q(v, t) x(v, t|q)] dv \right), \quad (9)$$

where $x(v, t|q)$ is the quantity of the intermediate good in industry v of quality q used in the production process.¹⁸ As standard in Schumpeterian growth models there is no storage technology for intermediate and final goods — produced intermediate goods are used in

¹⁶Since ventures either fail or succeed, and since the value of a venture conditional on succeeding is locally deterministic, both equity and debt contracts can achieve the same payoff profile, yielding either zero or a fraction of the successful venture's value. In practice, VCs and entrepreneurs typically hold a substantial fraction of start-ups' securities (see, e.g., Barry, Muscarella, Peavy, and Vetsuypens, 1990, Korteweg and Sorensen, 2010).

¹⁷This assumption is relevant only in cases when mixed strategies for industry expertise acquisition obtain, which will not be the case in the calibration. Otherwise, in the case of pure strategies, the equilibrium value of n_1 can be perfectly anticipated at the beginning of a period.

¹⁸Apart from the factor A_t , the specification for the final good production function (9) is standard in the Schumpeterian growth literature. See, e.g., Francois and Lloyd-Ellis (2003), and Klette and Kortum (2004).

final good production in the same period, and the output flow of the final good Y_t equals the flow of aggregate consumption C_t in equilibrium. The representation of the final good production function (9) already indicates that, in equilibrium, the best intermediate good in each industry, which is of quality $q(v, t)$, will be used in the production of the final good. The factor A_t in equation (9) follows the stochastic differential equation

$$\frac{dA_t}{A_t} = \delta(Z_t) dt + \sigma(Z_t) dB_t, \quad \text{with } A(0) > 0, \quad (10)$$

where B_t is a standard Brownian motion, and where the local drift δ and local risk exposure σ depend on the Markov state Z_t . The factor A_t allows the model to account for sources of aggregate growth and uncertainty other than creative destruction, which is essential for an empirically plausible calibration of the model.

2.7. Risk Sharing

Apart from the frictions described above, agents are free to trade financial claims in a frictionless Walrasian market where Arrow Debreu securities are in zero net supply. In Section 5, I relax this assumption and analyze the model's implications when risks associated with the performance-sensitive components of venture capital contracts cannot be shared by agents via side contracts.

3. Decentralized Equilibrium

In the following, I analyze symmetric perfect Bayesian Markov equilibria of the decentralized economy. When structural parameters differ across industries, a symmetric equilibrium refers to symmetry among all industries with identical parameters. Standard definitions of an allocation and a decentralized equilibrium are relegated to Appendix A.2.

Aggregate state variables in the economy are the Markov state Z and the level of aggregate output Y . Due to the iso-elastic properties of the setup, the economy scales by Y . Throughout, I will take advantage of the economy's scaling property and directly consider prices and compensation rates scaled by output Y . I mark these scaled variables by a tilde. For example, the blue-collar and white-collar wage-to-output ratios will be denoted by $\tilde{w}_B(Z)$ and $\tilde{w}_W(Z)$, respectively.

Below, I first analyze industry-level equilibrium conditions taking as given the drift and

volatility of aggregate consumption, denoted by $\mu(Z)$ and $\sigma(Z)$ respectively, the stochastic discount factor, denoted by ξ_t , and the blue-collar and white-collar wage-to-output ratios. Thereafter, I characterize the equilibrium conditions determining these endogenous aggregate variables.

3.1. Industry-level Quantities and Prices

3.1.1. Production

Producers of the final good behave competitively and take the prices of intermediate goods of various available qualities in each industry as given. The intermediate good producer with the highest-quality patent in an industry is the incumbent firm. Prices for intermediate goods, $p^x(v, t|q)$, are determined in the monopolistically competitive intermediate goods market. A final good producer's first-order condition yields the unit elastic intermediate good demand:

$$x(v, t|q) = \frac{Y_t}{p^x(v, t|q)}. \quad (11)$$

Given this intermediate good demand, the incumbent firm in an industry limits prices at the marginal cost of the previous incumbent who could enter using his inferior patent. If the previous incumbent's patent quality is inferior by a factor κ , then the profit-maximizing monopoly price for the highest quality intermediate good is given by

$$p^x(v, t|q) = \kappa \cdot w_B(Y_t, Z_t). \quad (12)$$

As a result, the incumbent firm with the highest-quality patent in industry v earns the following monopoly profits:

$$\pi_M(v, t) = (p^x(v, t|q) - w_B(Y_t, Z_t)) \cdot x(v, t|q) = \left(1 - \frac{1}{\kappa}\right) Y_t. \quad (13)$$

3.1.2. Venture Funding

Due to the above-mentioned scaling property of the economy, strategies determining the measures of ventures receiving funding (n_1 and n_2) do not depend on the current level of aggregate output Y . Yet venture investment denominated in final good consumption units will naturally depend on Y .

VC funding with industry expertise. First, consider the maximization problem of a VC fund that successfully acquired expertise in industry v , such that $\iota_1(v, t) = 1$. The maximum surplus this VC fund can attain by funding the most productive entrepreneur’s venture ideas is given by:

$$\tilde{\pi}_1(v, Z) = \max_{n_1 \in \mathbb{R}_0^+} \left\{ h_1(v, Z, n_1) \tilde{P}(v, Z) - (n_1 + c_l) \tilde{w}_W(Z) \right\}. \quad (14)$$

The VC fund takes market prices \tilde{P} and white-collar wage rates \tilde{w}_W as given and chooses the mass of ventures it funds, n_1 . When one of the ventures succeeds, which happens with Poisson intensity h_1 , a VC fund obtains revenues from selling the firm in an IPO or M&A transaction at the market price \tilde{P} . For expositional convenience I refer to the expected market value of these transactions as *VC-backed IPO volume*, and introduce the corresponding variable $\widetilde{ipo} \equiv \bar{h}_1 \tilde{P}$. On the other hand, the VC fund requires capital commitments from investors, which I define as $\widetilde{com} \equiv (n_1 + c_l) \tilde{w}_W$. These commitments are used to make venture investments, $n_1 \tilde{w}_W$, and to pay the part of VC fund managers’ compensation that rewards white-collar human capital, $c_l \tilde{w}_W$. This performance-insensitive part of fund manager compensation is often referred to as a “management fee” in practice. For states where $\widetilde{com} > 0$, I define $f^{man} \equiv \frac{c_l \tilde{w}_W}{\widetilde{com}}$ as the fraction of capital commitments that is used for this type of compensation.

The first-order condition associated with the maximization problem (14) yields the optimal quantity of ventures a VC fund with industry expertise funds:

$$n_1(v, Z) = \left(\eta \theta(v, Z) \frac{\tilde{P}(v, Z)}{\tilde{w}_W(Z)} \right)^{\frac{1}{1-\eta}}. \quad (15)$$

If no VC fund acquires the expertise needed to identify and support the most productive entrepreneur in a given industry ($\iota_1(v, t) = 0$), the entrepreneur’s venture ideas cannot be implemented, such that $n_1 = 0$.

Network VCs and industry expertise. A connected network VC chooses the probability with which she accepts the best available compensation offer from a VC fund and facilitates the fund’s acquisition of industry expertise. I denote the corresponding Markov strategy by $\bar{\iota}_1(v, Z_t) \equiv \Pr[\iota_1(v, t) = 1 | Z_t]$. Competition between VC funds implies that any positive surplus $\tilde{\pi}_1 > 0$, if it exists, will be appropriated by the connected network VC and the most productive entrepreneur, both of which are needed to implement the most

productive entrepreneur’s ideas. These two agents jointly obtain a corresponding venture ownership share $\frac{\tilde{\pi}_1}{\tilde{i}po}$, whenever $\tilde{i}po > 0$. The connected network VC and the most productive entrepreneur Nash bargain over this stake and obtain fractions ϱ and $(1 - \varrho)$ of it, respectively. The stake a network VC obtains represents a performance-sensitive fee that yields a payoff only in case a venture succeeds. This fee component is akin to the so-called “carry” fee in practice.¹⁹ For states where capital commitments are positive, let $f^{car} \equiv \frac{\varrho\tilde{\pi}_1}{com}$ denote the expected value of this performance-sensitive carry fee as a fraction of commitments.

The connected network VC will optimally facilitate expertise acquisition only when she can obtain expected carry fees $\varrho\tilde{\pi}_1$ that are weakly positive. Formally, $\bar{v}_1(v, Z)$ satisfies the following equilibrium conditions:

$$\begin{aligned} \bar{v}_1(v, Z) &= 0 && \text{if } \varrho\tilde{\pi}_1(v, Z) < 0, \\ \bar{v}_1(v, Z) &\in [0, 1] && \text{if } \varrho\tilde{\pi}_1(v, Z) = 0, \\ \bar{v}_1(v, Z) &= 1 && \text{if } \varrho\tilde{\pi}_1(v, Z) > 0. \end{aligned} \tag{16}$$

Economic profits from VC investments. Since VC funds with industry expertise may be able to generate economic profits on their investments it is worth clarifying to which extent VC fund investors obtain “abnormal” returns. Let npv denote the net present value of a VC fund’s investments before compensating VC fund managers, that is,

$$\widetilde{npv} \equiv \widetilde{i}po - (n_1\tilde{w}_W + (1 - \varrho)\tilde{\pi}_1). \tag{17}$$

A VC fund’s npv is given by the market value of funded ventures minus the sum of venture investment cost and the market value of the entrepreneur’s equity stake. I define α as the ratio of this npv to capital commitments, that is, $\alpha \equiv \frac{\widetilde{npv}}{com}$. This ratio may be viewed as a measure of abnormal returns or economic profits from VC investments, which are not compensation for exposures to aggregate risk.²⁰ The above results imply that, in equilibrium, the total value of management and carry fees equals the value of economic profits from investing. Thus, net of fees, fund investors obtain zero abnormal returns on their investments:

$$\alpha - f^{man} - f^{car} = 0. \tag{18}$$

¹⁹Consistent with carry fees in practice this fee is paid only when fund investors obtain a positive return on their investment.

²⁰Since this measure capitalizes economic profits as a net present value, it is more closely related to the generalized market equivalent measure proposed by Korteweg and Nagel (2014) than to the typical annualized alpha return measure.

As in Berk and Green’s (2004) partial equilibrium model of the mutual fund industry, competition for VC fund managers’ talent ensures that fund investors make zero “net-alpha” in equilibrium. This view is consistent with several empirical studies in the VC literature finding that returns to investors net of fees are approximately zero.²¹

Funding of regular venture ideas. The measure of regular venture ideas implemented by new or existing firms is determined by the free-entry condition

$$n_2(v, Z, n_1) = \max \left\{ n_2 \in \mathbb{R}_0^+ : \frac{h_2(v, Z, n_1, \phi n_2)}{n_2} \tilde{P}(v, Z) - \tilde{w}_W(Z) \geq 0 \right\}. \quad (19)$$

Condition (19) reflects that regular venture ideas are funded up to the point at which firms break even in expectation — the market value of the average regular venture equals the cost of venture idea implementation. The success intensity per regular venture is given by $\frac{h_2}{n_2}$. Each venture incurs cost \tilde{w}_W from compensating one unit of white-collar labor, which is needed to implement the venture idea. Whereas these agents obtain compensation commensurate with their outside options in the labor market, entrepreneurs do not obtain an extra surplus for generating *regular* venture ideas, as these ideas can be generated by any entrepreneur or firm, and are thus not scarce.

Following the optimal strategy (15), a VC fund with industry expertise extends funding to the most productive entrepreneur until the marginal benefit of implementing superior ideas is equal to the cost of implementation. Since regular venture ideas are strictly inferior to the most productive entrepreneur’s marginal venture idea, plugging (15) into condition (19) implies that no additional regular venture ideas will be funded ($n_2 = 0$) in periods when the most productive entrepreneur already obtained funding ($\iota_1 = 1$). Additional regular venture ideas are therefore funded only when no VC fund acquired industry expertise ($\iota_1 = 0$). In this case, condition (19) yields:

$$n_2(v, Z, 0) = \frac{1}{\phi} \left(\frac{\phi \theta(v, Z_t) \tilde{P}(v, Z_t)}{\tilde{w}_W(Z_t)} \right)^{\frac{1}{1-\eta}}. \quad (20)$$

Funding by existing firms. Firms solve a different optimization problem when deciding on the funding of ideas in product lines v where they already have the best patent. An incumbent takes into account that it cannibalizes its existing product. In contrast, firms that do not have the best product can steal business from the existing incumbent, implying

²¹See, e.g., Hall and Woodward (2007) and Korteweg and Nagel (2014).

that they have stronger private incentives to innovate.²² Thus, Arrow’s replacement effect, a typical feature of quality ladder models, applies — an incumbent firm optimally does not fund additional ventures in product lines where it already has the best patent (see a formal proof in Appendix A.3). An incumbent can, however, fund ventures to enter new product lines and steal business from other firms, as in the model of Klette and Kortum (2004).²³ In this case, incumbents solve the same maximization problem as all other entities funding regular ideas — the free entry condition (19) applies. Since it is not uniquely determined whether the measure n_2 of regular venture ideas is funded by established or new firms, and since the funding of regular ideas does not involve active VC intermediation, I will not interpret n_2 as VC-backed innovative activity going forward.

The above analysis leads to the following Proposition characterizing the equilibrium rate of creative destruction at the industry-level.

PROPOSITION 1 (Rate of creative destruction). *The expected rate of creative destruction at the industry level is given by:*

$$\bar{h}(v, Z) = \bar{\tau}_1(v, Z) h(v, n_1(v, Z)) + (1 - \bar{\tau}_1(v, Z)) h(v, Z, \phi n_2(v, Z, 0)), \quad (21)$$

where the Markov strategies $\bar{\tau}_1(v, Z_t)$, $n_1(v, Z_t)$, and $n_2(v, Z_t, 0)$ are characterized in (16), (15), and (20), respectively.

As VC firms’ endogenous intermediation decisions alter barriers to entry for the most productive entrepreneurs, they also affect the rate of creative destruction. $\bar{h}(v, Z)$ represents both the rate of new entry by firms into industries and the rate of exit by incumbents that are getting displaced by these new entrants. As entry poses a competitive threat, $\bar{h}(v, Z)$ has important effects on the prices and returns of ventures post IPO, as discussed in more detail in the next subsection.

3.1.3. Valuations in Competitive Financial Markets

Building on the results developed so far, I now characterize the market valuations of ventures that succeed in developing a new patent.

²²Greenwood and Jovanovic (1999) further argue that incumbents are at a relative disadvantage in implementing new technologies during times of technological revolutions.

²³Formally, this interpretation is feasible as long as each incumbent firm is restricted to have the best patents in a finite number of product lines.

PROPOSITION 2 (Market valuations). *The market price of a venture that successfully develops the new highest-quality intermediate good in industry v is:*

$$P(v, Y_t, Z_t) = \mathbb{E}_t \left[\int_t^{\tau^*} \frac{\xi_\tau}{\xi_t} \pi_M(v, \tau) d\tau \right] = \tilde{P}(v, Z_t) \cdot Y_t, \quad (22)$$

where $\tau^* \equiv \sup \{ \tau : M(v, \tau) = M(v, t) \}$, and where the profit flow rate $\pi_M(v, t)$ is given in equation (13). The Hamilton Jacobi Bellman (HJB) equation associated with (22) implies that the function $\tilde{P}(v, Z)$ solves the following system of equations for all $Z \in \Omega$:

$$\begin{aligned} 0 = & 1 - \frac{1}{\kappa} - (r_f(Z) + rp^D(v, Z) + rp^J(v, Z) + \bar{h}(v, Z) - \mu(Z)) \tilde{P}(v, Z) \\ & + \sum_{Z' \neq Z} \lambda_{ZZ'} \left(\tilde{P}(v, Z') - \tilde{P}(v, Z) \right). \end{aligned} \quad (23)$$

The expressions for the risk free rate r_f , the diffusion-risk premium rp^D , and the jump-risk premium rp^J are provided in Appendices A.3 and A.5.

Proof. See Appendices A.3 and A.5. ■

Proposition 2 highlights that the industry-specific rates of creative destruction, $\bar{h}(v, Z)$, have important implications for the market valuations of ventures that can undertake an IPO. A firm has a higher propensity of losing its new incumbent position to yet another entrant when the rate of entry is expected to stay persistently high. Since the rates of creative destruction also vary with the aggregate state Z , they further affect exposures to aggregate macroeconomic conditions and thus risk premia, in particular the jump-risk premium rp^J . As the VC industry facilitates entry of new firms through the provision of informed external finance, its equilibrium behavior affects these risk exposures. A firm's competitive expected return post IPO is given by the sum of the risk-free rate r_f and the diffusion and jump risk premia, rp^D and rp^J .

3.2. Aggregate Quantities and Prices

Output. The following proposition characterizes the equilibrium dynamics of aggregate output Y , which equals aggregate consumption C in equilibrium.

PROPOSITION 3 (Aggregate output). *Aggregate output follows the stochastic differential equation*

$$\frac{dY_t}{Y_t} = \mu(Z_t) dt + \sigma(Z_t) dB_t, \quad (24)$$

where the local drift μ takes the following form:

$$\mu(Z_t) = \delta(Z_t) + \log[\kappa] \int_{\Psi} \bar{h}(v, Z_t) dv. \quad (25)$$

Proof. See Appendix A.4. ■

Proposition 3 shows that the local drift of aggregate output, $\mu(Z_t)$, can be decomposed into the exogenous component $\delta(Z_t)$ and an endogenous component that depends on the equilibrium rates of creative destruction across industries, $\bar{h}(v, Z_t)$.

Stochastic discount factor. In the baseline model, agents can diversify idiosyncratic risks in equilibrium. The following Proposition characterizes the corresponding dynamics of the stochastic discount factor ξ_t .

PROPOSITION 4 (Stochastic discount factor). *The stochastic discount factor follows a Markov-modulated jump-diffusion,*

$$\frac{d\xi_t}{\xi_{t-}} = -r_f(Z_{t-}) dt - \gamma\sigma(Z_{t-}) dB_t + \sum_{Z' \neq Z_{t-}} (e^{\zeta(Z_{t-}, Z')} - 1) (dN_t(Z_{t-}, Z') - \lambda_{Z_{t-}, Z'} dt), \quad (26)$$

where the function $\zeta(Z, Z')$ is determined by agents' HJB equation provided in Appendix A.5.

Proof. See Appendix A.5. ■

In Section 5, I will analyze the model under the assumption of imperfect risk sharing.

Labor markets and wages. Combining the optimal intermediate good demand (11) and intermediate goods prices (12) with the fact that one unit of blue-collar labor produces one unit of the final good leads to the following market-clearing condition for blue-collar labor:

$$\int_{\Psi} x(v, t|q) dv = \frac{1}{\kappa \cdot \tilde{w}_B(Z_t)} = 1. \quad (27)$$

It follows that the blue-collar wage-to-output ratio is constant and given by $\tilde{w}_B = 1/\kappa$. The white-collar wage-to-output ratio $\tilde{w}_W(Z_t)$ solves the market clearing condition:

$$\int_{\Psi} [\bar{l}(v, Z) (n_1(v, Z) + c_i) + (1 - \bar{l}(v, Z)) n_2(v, Z, 0)] dv = 1. \quad (28)$$

The Markov strategies $\bar{t}_1(v, Z)$, $n_1(v, Z)$, and $n_2(v, Z, 0)$ are characterized in (16), (15), and (20) as functions of the white-collar wage rate \tilde{w}_W and the market price \tilde{P} . Thus, \tilde{w}_W varies across aggregate states Z , as the demand for white-collar labor from VC funds and innovative firms responds to forward-looking information encoded in market prices, \tilde{P} .

4. Calibration and Evaluation

In this section I calibrate the model and evaluate its predictions. The calibration aims to match the dynamics of a set of key variables relevant for macroeconomic dynamics, asset prices, and VC activity in U.S. data, such as consumption growth, risk free rates and risk prices, and the magnitude and cyclical of VC intermediation.

4.1. Choosing Parameters

Below, I first discuss preference parameters and aggregate consumption dynamics before addressing parameter choices relating to VC activity and endogenous growth. Table 1 reports the values of all state-invariant parameters of the model.

Table 1
State-invariant Parameters

Parameter	Variable	Value
Rate of time preference	β	0.015
Elasticity of intertemporal substitution	ψ	1.500
Coefficient of relative risk aversion	γ	7.500
Efficiency discount of regular venture ideas	ϕ	0.323
Jump size of the quality index per innovation	κ	1.148
Decreasing returns to scale parameter	η	0.585
White-collar labor required to acquire industry expertise ($v \in \Psi'$)	c_v	0.027
Network VC bargaining power	ρ	0.346
Mass of product lines potentially exhibiting active VC intermediation	$\int_{\Psi'} dv$	0.030

4.1.1. Consumption Dynamics and Preference Parameters

I parameterize the model based on a nine-state Markov chain for the aggregate state Z . The exogenous drift component of aggregate output growth, δ , and the local volatility, σ ,

are chosen so that the resulting equilibrium consumption process matches the one estimated in Chen (2010) state by state.²⁴ Throughout, I present state-dependent parameter values and equilibrium outcomes in three-by-three matrices that sort the nine Markov states along two dimensions: expected consumption growth, μ , and local volatility, σ (each categorized into *low*, *med*, and *high*). Table 2.a reports the values of δ and Table 5 in Appendix A.1 the values of μ and σ , as well as the unconditional probabilities of all states. It is worth noting that every Markov state Z is associated with a distinct drift $\mu(Z)$ — for example, there is still variation in the drift across states categorized as “high μ ” in the tables.

In addition, I choose the same preference parameters (β , ψ , and γ) as in Chen (2010). Together, these parameter choices discipline consumption dynamics and asset pricing moments of the economy and ensure reasonable aggregate risk premia and risk free rate dynamics, thus providing a good laboratory for jointly studying macroeconomic conditions and venture capital intermediation. Panels (a), (c), and (d) of Table 6 in Appendix A.1 illustrate asset pricing implications of this calibration by reporting the risk-free rate, jump-risk premia, and diffusion-risk premia. Following the asset pricing literature, I report risk premia of a levered claim to aggregate consumption, as considered for example in Abel (1999), Campbell (2003), and Bansal and Yaron (2004).

Table 2
State-dependent Aggregate Growth

Panels (a) and (b) report the exogenous and endogenous component of aggregate growth in each of the nine Markov states. Values are reported in percentage points.

(a) Exogenous drift: $\delta(Z)$				(b) Endogenous drift: $\log[\kappa] \int_{\Psi} \bar{h}(v, Z) dv$					
		σ					σ		
		low	med	high			low	med	high
	high	3.52	4.18	4.70		high	0.55	0.59	0.62
μ	med	1.35	1.36	1.38	μ	med	0.48	0.48	0.48
	low	-0.88	-1.53	-2.05		low	0.45	0.44	0.44

4.1.2. VC Activity and Endogenous Growth

The remaining parameters of the model relate to VC activity and the endogenous component of growth, which is due to creative destruction. VC activity is empirically concentrated

²⁴This process approximates the discrete-time model calibration in Bansal and Yaron (2004). I thank Hui Chen for providing the estimated Markov chain generator matrix.

in specific industries such as IT and Pharma,²⁵ and has historically been highly volatile and cyclical.²⁶ In light of these empirical regularities I consider a parsimonious parameterization of the economy with two types of industries: each industry $v \in \Psi$ belongs to one of two disjoint subsets Ψ' and Ψ'' . Across these two types of industries structural parameters differ along only two dimensions: first, the dynamics of productivity θ , and second, VC funds' cost parameter c_v . All other structural parameters are identical across industries. Industry heterogeneity is essential to capturing the concentration and cyclicity of VC activity, as cross-sectional allocations of human capital would be symmetric if all industries faced the exact same technological conditions at any point in time. I calibrate the model so that the set Ψ' represents industries that potentially exhibit active VC intermediation, whereas the set Ψ'' contains industries that do not. The mass of industries that potentially exhibit active VC intermediation, $\int_{\Psi'} dv$, is set to match the historical average ratio of capital commitments to the VC industry relative to aggregate consumption, which I estimate to be 0.3 percent based on data reported by the National Venture Capital Association (2012).²⁷ In the following, I discuss the remaining parameters of the model.

Innovation step size κ . The parameter κ is chosen to match the innovation step size estimate in Acemoglu, Akcigit, Bloom, and Kerr (2013), which implies choosing $\kappa = 1.148$. This parameter choice is also consistent with empirical evidence on IT sector markups estimated in Ward (2013).²⁸

Efficiency discount ϕ . I use Kortum and Lerner's (2000) patent production function estimates to choose the parameter ϕ , which measures the relative efficiency of regular and VC-backed innovative activity, and thus affects the social benefits of VC intermediation.²⁹

²⁵Kortum and Lerner (2000) find for example that the top three industries that obtain venture capital are drugs, office & computing, and communication equipment, constituting 54 percent of VC investments.

²⁶See, e.g., Gompers and Lerner (2004) and Gompers, Kovner, Lerner, and Scharfstein (2008).

²⁷My estimate is based on capital commitments to VC funds from 1990 to 2010 (National Venture Capital Association, 2012). I scale these numbers by U.S. final consumption expenditure in each year (The World Bank, 2012). To estimate the expected value I compute the average of this time-series. The calibrated model matches this number, that is, $\mathbb{E}\{\int_{\Psi'} \widetilde{com}(v, Z) dv\} = 0.3$ percent.

²⁸In the model the parameter κ pins down the profits-to-sales ratio of an incumbent firm, which is given by $(1 - 1/\kappa)$. Ward (2013) estimates average aggregate IT sector markups from 1972 – 2012 and finds that $1/(1 - \frac{EBITDA}{Sales}) - 1 = 0.142$. In my model this markup is obtained by choosing $\kappa = 1.142$.

²⁹Kortum and Lerner (2000) estimate a patent production function $P = (R + bV)^\alpha$, where in their specification P refers to patenting, R measures R&D expenditures not funded by VC firms, and V is R&D funded by VC firms. Based on a variety of empirical tests, the authors obtain an average estimate of the parameter b of 3.1 (see footnote 26 in Kortum and Lerner (2000)). This estimate corresponds to a value of $1/3.1$ for the parameter ϕ in my model.

White-collar VC manager input c_i . The parameter c_i applying to industries Ψ' , which may exhibit active intermediation, influences the magnitude of VC *management* fees. I set this parameter so that average management fees amount to 3 percent of invested capital, targeting empirical findings by Hall and Woodward (2007).³⁰ Higher values of c_i effectively increase the strike price of the option to provide active VC intermediation, thus making it a less likely equilibrium outcome. For parsimoniousness, I assume that in industries Ψ'' the information cost parameter c_i is high enough to preclude active VC intermediation, consistent with the above-discussed classification of these industries.

Decreasing returns to scale parameter η . In the model, the combined payoffs of VC managers and entrepreneurs are tightly linked to the decreasing returns to scale parameter η , as this parameter influences the magnitude of rents from active VC intermediation. Hall and Woodward (2007) (p. 21) find that general partners' total earnings (management plus carry fees) are 26 percent of funds invested. Hall and Woodward (2007) further estimate that whereas entrepreneurs historically earned an average of \$9.2 million (in 2006 dollars) from each company that attracted VC funding, VC managers received an average of \$5.5 million in fee revenue from each company they backed. I set the parameter η so that in equilibrium the ratio of the sum of VC fees and entrepreneurial payoffs to capital commitments is consistent with these empirical estimates, which leads me to choose $\eta = 0.585$.³¹ This parameter value also lies within one standard deviation of the point estimate Kortum and Lerner (2000) obtain for the decreasing returns to scale parameter of their patent production function.³²

VC bargaining power ϱ . In the model, VC carry fees and entrepreneurial payoffs are bargained fractions ϱ and $(1 - \varrho)$ of the rents $\tilde{\pi}_1$, respectively. I choose the parameter ϱ so that the ratio of carry fees to entrepreneurial payoffs corresponds to the estimates by Hall and Woodward (2007) mentioned above.³³ Overall, the calibration thus ensures that the model matches aggregate VC carry fees, VC management fees, and entrepreneurial payoffs.³⁴

³⁰In the calibrated model $\mathbb{E}[f^{man} | \widehat{com} > 0] = 0.03$ in industries $v \in \Psi'$.

³¹The total payoffs to VC managers and entrepreneurs as a fraction of capital commitments is thus $0.26 \cdot (1 + \frac{9.2}{5.5}) \approx 0.69$. The calibrated model yields: $\mathbb{E}[f^{man} + f^{car} + \frac{(1-\varrho)\tilde{\pi}_1}{com} | \widehat{com} > 0] = 0.69$.

³²When using an instrumental variable approach, Kortum and Lerner (2000) estimate a decreasing returns to scale parameter of about 0.52 (see Panel B of Table 4 in Kortum and Lerner (2000)).

³³The ratio of total VC fees to the value of payoffs received by entrepreneurs is $5.5/9.2$. Since carry fees account for a fraction $0.23/0.26$ of total VC fees, the ratio of carry fee payoffs to entrepreneurial payoffs is approximately $(5.5 \cdot 0.23/0.26)/9.2$, which corresponds to the ratio of bargaining powers in the model, $\frac{\varrho}{1-\varrho}$, and yields $\varrho = 0.346$.

³⁴On average, carry fees represent 23 percent of provided capital, as estimated by Hall and Woodward (2007). In the calibrated model $\mathbb{E}[f^{car} | \widehat{com} > 0] = 0.23$ in industries $v \in \Psi'$.

Productivity θ . The only remaining state-dependent parameter of the model is productivity θ . The productivity processes in the two sets of industries Ψ' and Ψ'' have different means and are specified as differentially exposed to macroeconomic conditions, as measured by the state-contingent consumption growth drifts estimated in Chen (2010). For industries Ψ' , I specify a non-linear relation that ensures that the model matches the highly cyclical and skewed distribution of aggregate commitments to VC funds in the data. The caption of Table 3 reports this relation as well as the corresponding state-contingent values of θ . The specification is characterized by three parameters relating to the mean, the volatility, and the skewness of the productivity distribution. Mean productivity is set so that industries Ψ' exhibit an average rate of creative destruction that is representative of the types of sectors that receive VC funding. In particular, using estimates from Caballero and Jaffe (1993), I match an average rate of creative destruction of 8 percent.³⁵ The remaining two parameters are set so that the volatility and the peak levels of aggregate capital commitments to VC funds in the model match their data counterparts.³⁶ Empirical peak levels were reached at the end of the internet boom, when aggregate commitments scaled by aggregate consumption were more than three standard deviations above their average value. The proposed specification also replicates the strong correlation between VC activity and aggregate conditions, consistent with existing empirical evidence.³⁷

For industries Ψ'' , I consider a simple linear specification for θ that is characterized by two parameters determining average productivity and the exposure to macroeconomic conditions (see caption of Table 7 in Appendix A.1). I set the mean of θ in industries Ψ'' so that the average rates of creative destruction across all industries Ψ matches the average rate of creative destruction in the U.S. economy of 4 percent per year, as estimated by Caballero and Jaffe (1993). The exposure parameter is set so that the standard deviation of white-collar wage-to-output ratios relative to their mean, $\frac{Std[\tilde{w}]}{E[\tilde{w}]}$, is consistent with empirical estimates obtained from income shares data for the top 5 percent of income earners in the United States.³⁸ White-collar wage-to-output ratios are reported in Table 6.b. I associate

³⁵This number is the average estimated rate of creative destruction across the sectors “Drugs,” “Computers and data processing,” “Electronic communications,” and “Medical” reported in Table 5 of Caballero and Jaffe (1993).

³⁶Scaled commitments are measured as discussed in footnote 27. The standard deviation of these empirical values is 0.3 percent. Correspondingly, in the calibrated model $Std[\widetilde{com}] = 0.3$ percent. The historical peak level of annual commitments was 1.3 percent of U.S. final consumption expenditure, and was reached in the year 2000. In the model this level is reached in the high- μ /high- σ state (see Table 3.b).

³⁷See, e.g., Gompers and Lerner (1998), Jeng and Wells (2000), Kaplan and Schoar (2005), and Gompers, Kovner, Lerner, and Scharfstein (2008).

³⁸I use data provided by Alvaredo, Atkinson, Piketty, and Saez (2012). After de-trending income shares for the top 5 percent (incl. capital gains), I find that the volatility of income shares relative to their mean was 5.5 percent during the time period from 1990 to 2010. I focus on this time period, since top income and

income fluctuations for white-collar labor with those for top income earners, as agents from this group are likely to be recruited for innovation- and finance-related jobs in practice.³⁹ An important feature captured by the calibration is the empirical fact that top earners' income share has substantial positive comovement with the business cycle (see Piketty and Saez, 2003, Parker and Vissing-Jorgensen, 2009, 2010). Consistent with this observation, white-collar human capital, which is essential to economic growth, can appropriate a larger fraction of aggregate output in states with high aggregate growth (see high μ states in Table 6.b).

4.2. Results of the Calibration

In this section, I analyze the implications of the calibrated model. First, I discuss aggregate measures of VC intermediation and evaluate the impact of VC investment on the macroeconomy. Second, I analyze the payoffs and returns to various parties involved in the VC process. Tables 3 and 4 report outcomes in VC-backed industries Ψ' in each of the nine Markov states.

4.2.1. The Aggregate Impact of VC Intermediation

Capital commitments. Table 3.a indicates the states in which VC funds provide active intermediation. Table 3.b reports the capital commitments the VC industry obtains as a fraction of aggregate consumption. In boom states (high μ states), the VC industry attracts larger commitments, with peak levels reaching magnitudes comparable to those observed during the internet boom. In contrast, in recessions (low μ states), VC funds are unable to attract funds as rents from active intermediation are insufficient to cover management fees. As a result, active VC intermediation breaks down.

VC-backed IPO and M&A volume. Table 3.d reports the aggregate market value of VC-backed IPOs and M&A transactions scaled by aggregate consumption.⁴⁰ Consistent with the findings of the empirical literature the model features large booms and busts in

wage shares display a U-shaped pattern over the last century (see Piketty and Saez, 2003). Since the 1980s there has been an upward trend in the income share of the top 1 percent and the top 5 percent.

³⁹Naturally, there is no single percentile cut-off in the income space that provides a clear-cut match to income for white-collar labor in the model. In this regard, it is worth noting that empirically, the income share of the top 1 percent also drives much of the variation in the top 5 percent.

⁴⁰As noted in Section 3.1.2, successful ventures can either undertake an IPO or can be acquired by an existing firm.

Table 3State-contingent Outcomes in VC-Backed Industries Ψ'

The panels report state-contingent outcomes in VC-backed industries Ψ' . Panel (a) indicates in which states VC firms provide active intermediation. Panel (b) reports capital commitments to VC funds scaled by aggregate consumption. Panels (c) and (d) show the rates of creative destruction and the aggregate value of VC-backed IPOs and acquisitions scaled by aggregate consumption. Panels (e) and (f) report productivity and the ratio of entry rates to productivity. The state-contingent productivity values follow from the calibrated relation:

$$\theta(v, Z) = a + b \cdot (\mu(Z) - \min_{\forall Z} \{\mu(Z)\})^c, \quad (29)$$

where, $a = 0.0063$, $b = 24.53$, $c = 1.85$, and $\mu(Z)$ are the aggregate growth drifts from Chen (2010).

(a) Active VC intermediation: $\bar{t}_1(v, Z)$					(b) Capital commitments: $\int_{\Psi'} \widetilde{com}(v, Z) dv$				
		σ					σ		
		low	med	high			low	med	high
	high	1	1	1		high	0.008	0.010	0.013
μ	med	1	1	1	μ	med	0.002	0.002	0.002
	low	0	0	0		low	0.000	0.000	0.000

(c) Rate of creative destruction: $\bar{h}(v, Z)$					(d) VC-backed IPOs: $\int_{\Psi'} \widetilde{ipo}(v, Z) dv$				
		σ					σ		
		low	med	high			low	med	high
	high	0.223	0.313	0.402		high	0.014	0.018	0.021
μ	med	0.045	0.047	0.048	μ	med	0.004	0.004	0.004
	low	0.001	0.000	0.000		low	0.000	0.000	0.000

(e) Productivity: $\theta(v, Z)$					(f) Entry-to-productivity ratio: $\frac{\bar{h}(v, Z)}{\theta(v, Z)}$				
		σ					σ		
		low	med	high			low	med	high
	high	0.129	0.157	0.183		high	1.733	1.989	2.200
μ	med	0.054	0.055	0.055	μ	med	0.830	0.851	0.870
	low	0.013	0.008	0.006		low	0.062	0.032	0.024

transaction volume that are highly correlated with commitments to VC funds and aggregate conditions.⁴¹ The model provides a quite remarkable fit of empirical moments relating to IPO and M&A volume, which were not targeted in the calibration. On average, transactions amount to 0.5 percent of aggregate consumption in the model, whereas in the data the average is 0.6 percent.⁴² The standard deviation is 0.6 percent both in the data and in the model. Aggregate transactions peaked during the internet boom at 2.4 percent, and reach 2.1 percent in the model. This close match validates the model’s representation of how the VC industry adds value by converting capital commitments into successful IPOs and acquisitions. In addition, as the calibration matches the payoffs to entrepreneurs and VCs, this result also implies a good representation of the payoffs to VC investors, which are the third party receiving payoffs from IPOs and M&A transactions.

Creative destruction. Rates of creative destruction \bar{h} , reported in Table 3.c, spike during booms when many entrants obtain finance from VC funds. As a result, even those ventures that successfully enter the market in booms are exposed to high failure risk associated with the displacement by new entrants. The dynamics of creative destruction are tightly linked to those of IPO volume, consistent with the findings of Jovanovic and Rousseau (2003, 2005), who document spiking rates of entry and creative destruction as central characteristics of technological revolutions. The arrival rate of innovations \bar{h} is highly cyclical not only because of exogenous variation in θ , but also because venture investment endogenously collapses in recession states, when new entrants do not receive funding from active VC funds (see Tables 3.e and 3.f).

The aggregate impact of VC investment. The calibrated model is a natural laboratory to evaluate the effect of VC investment on the aggregate economy. In states with active VC intermediation (see Table 3.a) VC investments contribute an endogenous fraction of aggregate growth. These growth contributions are given by the product of the state-contingent entry rates \bar{h} reported in Table 3.c and $\log[\kappa] \int_{\Psi'} dv \approx 0.0041$. This calculation implies an average growth contribution of about 3 to 4 basis points per annum. We know from Lucas (1987) type calculations that even small increases in growth are very valuable. Indeed, the calibrated model predicts that agents would be willing to give up between 1 and

⁴¹See, e.g., Gompers and Lerner (1998) and Jeng and Wells (2000).

⁴²Estimates are based on data published by the National Venture Capital Association (2012) for the time period from 1990 to 2010. I add the “Post Offer Value” of venture-backed IPOs in Figure 5.03 to prices of known transactions from venture-backed Merger & Acquisitions reported in Figure 5.07. I scale the sum of these two valuations by U.S. final consumption expenditure (The World Bank, 2012) in each year.

2 percent of lifetime consumption to avoid losing the growth created by VC investments.⁴³ These magnitudes are significant, in particular when compared to estimates of the cost of business cycles (see Lucas (2003) for an overview). Alvarez and Jermann (2004) estimate for example that the benefit of eliminating consumption fluctuations corresponding to business cycle frequencies is at most 0.49 percent of lifetime consumption. The results highlight the potential importance of policies affecting the VC industry, which obtained significant capital commitments only after regulatory reforms and tax changes were passed in the late 1970s and early 1980s.⁴⁴

The extreme cyclicity of VC activity has a large impact on the above estimates, as agents are averse to persistent fluctuations in growth. To determine the magnitude of the risk discount agents apply I also perform the above-described calculation under the counterfactual where VC investments create the same average growth contributions but without any fluctuations. Agents would be willing to give up about 3 percent of lifetime consumption for these constant growth contributions, indicating a very significant risk discount embedded in the above estimates. In Section 5, I will further analyze how the results are affected when entrepreneurs and VC managers cannot share idiosyncratic venture risks through financial markets.

The value agents assign to VC investments' aggregate growth contributions significantly exceeds actual levels of VC fund commitments, which, as matched by the calibration, were historically on average around 0.3 percent of aggregate consumption and peaked at 1.3 percent during the internet boom. Despite the extreme cyclicity, the model thus indicates a potent positive societal impact of VC activity. VC investments create significant positive externalities: their innovations have a permanent positive effect by setting the stage for future innovations — as suggested by Newton, innovators stand “on the shoulders of giants.” Yet, due to competition, firms obtain profits from patents only for a limited time period, which can lead private parties to insufficiently internalize the social benefits of innovative activity. This force is particularly strong when competition between new entrants is fierce, as for example during a venture capital boom.

⁴³This calculation is performed for the unconditional expectation of the value function, $\mathbb{E}[J]$, starting from an allocation where agents hold claims to constant fractions of aggregate consumption.

⁴⁴See footnote 5 for additional details.

4.2.2. Payoffs to VCs and Entrepreneurs

In this section I evaluate the model’s predictions for payoffs to VC managers and entrepreneurs.

VC fees. Panels (a) and (b) of Table 4 report management fees and expected carry fees as fractions of commitments. Whereas management fees are paid with certainty using fund commitments, carry fees are stochastic at the fund-level and are only paid when a VC fund generates profits from bringing ventures to the market. As discussed above, fees are set such that fund investors obtain zero “net alpha” in equilibrium (as in Berk and Green, 2004) — thus, fees correspond to what is sometimes referred to as “gross alpha” in the literature. VC funds generate economic profits at the time when new ventures are selected and are still subject to purely idiosyncratic success risk. Total fees are the product of commitments reported in Table 3.b and $(f^{man} + f^{car})$. While the model is calibrated to match the average level of VC fees relative to contributed capital, it predicts strongly pro-cyclical variation in total fee revenues, consistent with findings of the empirical literature.⁴⁵ In addition, in booms, performance-sensitive carry fees represent a larger share of total fees.

Payoffs to entrepreneurs. High idiosyncratic risk and highly skewed return distributions are key features of venture capital data (see, e.g., Hall and Woodward, 2007, Kaplan and Lerner, 2010). The model replicates this fact — the vast majority of ventures that obtain funding fail, while a few obtain large payoffs from IPOs or M&A transactions. Since entrepreneurs are compensated with venture ownership stakes their payoffs are also exposed to this extreme idiosyncratic risk. While in the baseline model this risk can be shared via separate side contracts, I analyze in Section 5 the model’s implications when this type of risk sharing is infeasible. Nash bargaining weights determine the relative amounts of venture equity backing entrepreneurial compensation and VC carry fees. In particular, the calibrated VC bargaining power parameter $\rho = 0.346$ implies that aggregate entrepreneurial payoffs in each state are about twice as large as aggregate carry fees, which are given by $\widetilde{com} \cdot f^{car}$. Aggregate payoffs to entrepreneurs are thus also highly cyclical, just like aggregate carry fee revenues and VC-backed IPO volume.

⁴⁵See, e.g., Ljungqvist and Richardson (2003); Kaplan and Schoar (2005); Gompers, Kovner, Lerner, and Scharfstein (2008); Phalippou and Gottschalg (2009); Korteweg and Sorensen (2010).

Table 4
VC Fees and Risk Premia in VC-Backed Industries Ψ'

Panels (a) and (b) list VC management fees and expected carry fee revenues as fractions of VC fund commitments. Panels (c) and (d) report jump-risk premia and diffusion-risk premia in each of the nine Markov states.

(a) VC management fees: $f^{man}(v, Z)$					(b) VC carry fees: $f^{car}(v, Z)$				
		σ					σ		
		low	med	high			low	med	high
μ	high	0.010	0.008	0.007	μ	high	0.239	0.240	0.241
	med	0.036	0.034	0.033		med	0.224	0.225	0.226
	low	0.000	0.000	0.000		low	0.000	0.000	0.000
(c) Jump-risk premium: $rp^J(v, Z)$					(d) Diffusion-risk premium: $rp^D(v, Z)$				
		σ					σ		
		low	med	high			low	med	high
μ	high	-0.056	-0.097	-0.141	μ	high	0.003	0.005	0.008
	med	-0.014	-0.021	-0.026		med	0.003	0.005	0.008
	low	-0.016	-0.019	-0.022		low	0.003	0.005	0.008

4.2.3. Expected Post-IPO Returns

Whereas venture success risk is purely idiosyncratic at the implementation stage, the cash flows that a venture generates conditional on a success are exposed to aggregate risk. Ventures that are sold in competitive markets, for example in an IPO, thus earn a risk premium. This risk premium is the sum of a jump-risk premium rp^J and a diffusion-risk premium rp^D , which compensate investors for exposures to Markov state shocks dZ and Brownian shocks dB , respectively. These two risk premia components are reported in Panels (c) and (d) of Table 4.

Diffusion risk premia are identical across industries and small in magnitude, just like those for claims to aggregate consumption.⁴⁶ In contrast, jump-risk premia are affected by the industry-specific dynamics of creative destruction, which depend on both productivity and venture funding. These jump-risk premia are strongly counter-cyclical and negative, pushing down expected post-IPO returns, which are given by the sum of the two risk premia

⁴⁶All incumbents have a beta of one with respect to aggregate Brownian shocks dB .

components and the risk-free rate r_f reported in Table 6.a. Expected returns are particularly low during booms (high μ states), when they range between minus 1.5 percent and minus 9.3 percent. Thus, whereas VC funds' profits and fees spike in booms, reaching about 25% of committed capital, investors' expected post-IPO returns become locally negative.

These low discount rates imply high IPO and acquisition valuations relative to expected future dividends, creating the appearance of a “bubble.” High valuations in turn direct large amounts of capital to VC funds. Endogenous declines in discount rates in booms thus play an essential role in rationalizing “lenient” funding standards, in the sense that marginal VC investments yield on average low future dividends. Time-varying funding standards and apparent valuation bubbles are central characteristics of the venture capital “boom-bust-cycle,” which has received significant attention in the literature.⁴⁷ While some of the existing literature argues that the strong negative correlation between VC fund inflows and future returns indicates investor irrationality (e.g., Gompers and Lerner, 2003) or increased competition between investors in booms (e.g., Gompers and Lerner, 2000), my quantitative model shows that this pattern also emerges in an environment with rational investors that obtain competitive returns after fees.

The fact that jump-risk premia are lowest in booms indicates that firms in VC-backed industries Ψ' are particularly good hedges against *low-frequency* risks (shocks dZ) in these times. In booms, households are worried about changes in the state Z that would lead to a persistent slow-down in aggregate growth. However, such shocks are partially good news for ventures that already attracted funding and successfully entered the market during a boom. Although their dividend growth suffers when aggregate growth declines, these incumbents benefit to a first-order degree from a decrease in entry by additional competitors. As venture funding for new firms dries up in downturns and rates of creative destruction plummet, incumbents face a lower threat of displacement, making them relatively safer.

This effect on risk premia is naturally strongest in exactly those industries that have the highest levels of IPOs in booms and exhibit the most extreme busts in entry in ensuing downturns, a pattern typical for young growth industries such as the IT industry during the internet boom. Pro-cyclical entry is thus self-reinforcing through a risk premium channel: pro-cyclical entry lowers discount rates in booms and raises venture funding in these times, which in turn makes entry more pro-cyclical.

⁴⁷See, e.g., Gompers and Lerner (1998), Jeng and Wells (2000), Gompers and Lerner (2003), and Korteweg and Sorensen (2010). Kaplan and Lerner (2010) conclude that “...there is a strong negative correlation between VC returns and the preceding years' capital commitments and investments.” Poor post-IPO return performance is also consistent with the empirical phenomenon known as the “net issues puzzle” (Loughran and Ritter, 1995).

The empirical literature lends support to the notion that product market entry and increased IPO volume have a detrimental effect on industry incumbents’ stock prices and operating performance (see, e.g. Hsu, Reed, and Rocholl, 2010). IPO volume and VC funding are further strongly related and pro-cyclical.⁴⁸ Finally, if one interprets ventures that undertake an IPO in VC-backed industries Ψ' as “growth firms” in the sense of the asset pricing literature, then existing empirical evidence lends support to the model’s prediction that these types of firms indeed have lower exposures to low-frequency risks and therefore low risk premia.⁴⁹ While these empirical observations are consistent with the mechanism and the predictions of the model, I discuss in the next subsection several empirical challenges associated with estimating standard risk-based asset pricing models in an environment as the one proposed in this paper.

4.3. Econometric Challenges

The presented model conceptualizes at least four challenges econometricians face when applying standard risk-based asset pricing models in the context of venture capital, adding to significant measurement and selection problems with VC data.⁵⁰

First, a typical assumption of standard models, such as the CAPM or the consumption-based capital asset pricing model (CCAPM), are perfectly competitive financial markets. Yet, in the case of venture capital, investors exhibit strong heterogeneity with regards to their access to information and investment opportunities. This heterogeneity implies that VC firms may have the ability to make economic profits on their investments. It is generally difficult to cleanly empirically differentiate this profit component from the risk premium component, in particular since the two components have inverse cyclical patterns — whereas abnormal returns peak in booms, conditional risk premia plummet — and since available VC data typically does not provide reliable market valuations at standard frequencies.⁵¹

Second, valuations are not only affected by high frequency shocks (dB) but also by low-frequency shocks (dZ), implying that estimating a single market beta picks up an amalga-

⁴⁸See, e.g., Gompers and Lerner (1998), Jeng and Wells (2000), and Gompers, Kovner, Lerner, and Scharfstein (2008).

⁴⁹See, e.g., Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008), and Bansal, Kiku, and Yaron (2012).

⁵⁰See, e.g., Hwang, Quigley, and Woodward (2005), Korteweg and Sorensen (2010), and Hall and Woodward (2007, 2010) for papers providing empirical approaches to address selection and measurement problems with VC data.

⁵¹VC firms have discretion over setting and reporting valuations over time, which also causes the above-mentioned selection problems.

mation of two risk exposures. In fact, Tables 4.c and 4.d indicate that the two risk premium components associated with high- and low-frequency shocks have opposite signs and opposite cyclical dynamics. As the betas are further correlated with variation in market risk premia and volatility, estimating unconditional betas also leads to biases.⁵²

Third, while precisely estimating exposures to low-frequency risks requires long time-series of data, VC data spans only a relatively short time period, since the industry started to attract significant commitments only in the last two to three decades.

Fourth, the model captures the fact that the stock market systematically mismeasures the total wealth portfolio, since it consists of incumbent firms only and misses the value of future entrants. Using a market index such as the S&P 500 as a proxy for the total wealth portfolio can thus be particularly problematic in times when a large part of future growth is expected to be generated by future entrants.

5. Imperfect Risk Sharing

The analysis thus far assumed that agents can perfectly share risks. In particular, entrepreneurs and network VCs were able to insure idiosyncratic risks associated with performance-sensitive compensation by selling state-contingent contractual claims in financial markets. In this section, I explore the model's implications when financial markets are instead incomplete in that risk sharing of this type is infeasible. Exposures to idiosyncratic risks can for example emerge as optimal contractual arrangements in the presence of agency problems,⁵³ which are likely a first-order concern in venture capital.

A convenient feature of the proposed setup is that the analysis of the general equilibrium economy remains tractable in the presence of this market incompleteness, and does not require additional simplifying assumptions. Entrepreneurs and network VCs cannot enter *ex ante* contracts that specify payments conditional on the future realizations of their own venture capital opportunities (network connections or superior ideas), and thus, stay exposed to uninsurable idiosyncratic income risks. I provide a technical analysis in Appendix A.6, and describe the key results of the analysis in this section.

⁵²See, e.g., Grant (1977) and Jagannathan and Wang (1996).

⁵³See Admati and Pfleiderer (1994) and Holmstrom and Tirole (1997). See also Schmidt (2003) and Casamatta (2003) for papers that consider optimal security design in the context of venture capital.

Consumption and portfolio choices. Due to the presence of undiversifiable idiosyncratic income shocks, the economy features dispersion in consumption growth across agents. While solving economies of this type typically requires approximation techniques or computationally-intensive methods, analyzing the proposed model under incomplete risk-sharing remains tractable. This result is in part due to the extreme nature of idiosyncratic risk faced by agents involved in venture capital in the model. Large idiosyncratic uncertainty, in particular for entrepreneurs, is in turn an important empirical regularity, as emphasized by Hall and Woodward (2010). While most VC-backed entrepreneurs don’t receive any payoffs associated with IPOs or acquisitions, very few receive over a billion dollars. Because of this large idiosyncratic risk, Hall and Woodward (2010) find that, “an entrepreneur with a coefficient of relative risk aversion of two places a certainty equivalent value only slightly greater than zero” on the interests in her own company. A similar result obtains in the presented model. Here, the extreme discount applied to future risky payoffs from VC contracts implies that the dynamic portfolio optimization problems of agents involved in the VC process become similar to those of agents that do not expect to receive performance-sensitive venture payoffs. Constrained optimal risk sharing then implies that at any point in time, all agents optimally hold the diversified market portfolio of tradable securities and choose the same consumption-to-tradable-wealth ratios.⁵⁴ As a result, the cross-sectional wealth distribution does not emerge as a state variable — the only aggregate state variables are still Y and Z , just as in the baseline analysis.

Consumption dynamics and asset prices. I now discuss how consumption dynamics and asset prices differ from the baseline model with complete risk sharing. Let $\tilde{P}_T(t)$ denote the market price of the portfolio of all tradable assets that are in positive net-supply at time t , also referred to as the *market portfolio*. Recall that new entrants are created through the joint efforts of entrepreneurs, network VCs, and white-collar labor. Correspondingly, in every period, the total equilibrium compensation this group of agents obtains is exactly equal to the total market value of the new entrants. A time-varying, endogenous fraction of this total market value is granted as performance-sensitive compensation to entrepreneurs and network VCs, and claims to this compensation cannot be traded until the idiosyncratic success risk of ventures is realized (e.g., an IPO occurs). The remaining, complementary fraction of total entrant equity is granted to diversified investors in exchange for the upfront capital they provide, which is used to compensate white-collar labor.

⁵⁴A similar result obtains in the asset pricing model of Garleanu, Panageas, Papanikolaou, and Yu (2015) who consider a setting with extreme idiosyncratic risks in order to study the effects of imperfect risk sharing on the value premium.

Let $\varpi_j(t)$ denote the share of the tradable market portfolio owned by agent $j = (S, i_G, i_S)$ at time t . The aggregate wealth gain obtained by idiosyncratically successful entrepreneurs and network VCs is given by:

$$\int_{\Psi} \bar{v}_1(v, Z) \tilde{\pi}_1(v, Z) dv dt, \quad (30)$$

where $\bar{v}_1(v, Z)$ and $\tilde{\pi}_1(v, Z)$ were defined in Section 3. The concentration of these wealth gains among a subset of agents implies that, in relative terms, all other agents in the economy become poorer — almost all agents’ wealth shares decline, but a few agents obtain very large gains. As all agents optimally choose identical consumption-to-tradable-wealth ratios, each agent’s consumption growth can be written as the sum of aggregate output growth, $\frac{dY}{Y}$, and the growth in the agent’s ownership share, $\frac{d\varpi_j}{\varpi_j}$. Since only a measure zero of agents obtain large wealth gains from venture capital in each period, an agent’s consumption growth is almost surely given by:

$$\frac{dC_j(t)}{C_j(t)} = \mu(Z_t)dt + \sigma(Z_t)dB - \frac{\int_{\Psi} \bar{v}_1(v, Z_t) \tilde{\pi}_1(v, Z_t) dv}{\tilde{P}_T(Z_t)} dt. \quad (31)$$

As shown in Appendix A.6, the equilibrium market prices of all assets that are *tradable* are set as if the dynamics for all agents’ marginal utility was governed by the consumption dynamics displayed in equation (31). Relative to the economy with perfect risk sharing these consumption dynamics are perturbed by a displacement term: almost all agents’ consumption growth is dampened, relative to perfect risk sharing, as a small subset of entrepreneurs and VCs obtain disproportionately large wealth gains.⁵⁵ This displacement term is given by the ratio of the aggregate uninsurable wealth gains by VCs and entrepreneurs displayed in (30) to the value of the market portfolio of tradable assets, \tilde{P}_T .⁵⁶

Market prices and allocations. To gauge the impact of imperfect risk sharing on the quantitative results discussed in the previous sections, I re-solve the model with the param-

⁵⁵Several recent asset pricing papers feature related concepts of displacement risk in endowment economies with market incompleteness and highlight that this type of risk can help address asset pricing phenomena such as the value premium (see Garleanu, Kogan, and Panageas, 2012, Kogan, Papanikolaou, and Stoffman, 2015, Garleanu, Panageas, Papanikolaou, and Yu, 2015). Kogan and Papanikolaou (2014) show that preferences with “keeping up with the Joneses” motives can amplify the asset pricing implications of uninsurable wealth gains.

⁵⁶This displacement effect implies that the aggregate wealth share of the group of entrepreneurs and network VCs converges to one in the long run. The wealth distribution is, however, inconsequential for the equilibrium dynamics of the variables of interest in this study. Therefore, I refrain from introducing additional model features, such as overlapping generations.

ter values chosen in Section 4.1.⁵⁷ A convenient feature of the model is that all industry-level equilibrium conditions presented in Section 3.1 and the aggregate output dynamics and labor market clearing conditions in Section 3.2 still hold in the setting with imperfect risk sharing. Solutions to these equilibrium conditions are, however, affected by the fact that consumption dynamics and the corresponding state pricing process ξ are different under incomplete risk sharing. In particular, as discussed above, the SDF dynamics relevant for tradable assets are effectively governed by the perturbed consumption growth dynamics displayed in equation (31). I find that the perturbations due to the highlighted displacement term are quantitatively small, since the value of venture equity obtained by entrepreneurs and VCs is small relative to the value of the stock of all tradable assets in the economy. The average value of the displacement term is 0.25 basis points and the maximum value is less than 1 basis point. As the total wealth gains of VCs and entrepreneurs are pro-cyclical, imperfect risk sharing has a very minor dampening effect on the cyclicalities of almost all agents' consumption growth. The dynamics of the state pricing process therefore also remain very similar when compared to the baseline analysis. Facing almost identical market prices, agents and firms also behave almost the same way. Consequently, the model with imperfect risk sharing is still consistent with the moments targeted in Section 4.1 and generates quantitatively similar predictions as discussed in Section 4.2.

Agent utility and value of aggregate VC investment. Despite these similarities in aggregate outcomes, entrepreneurs and venture capitalists lose very significantly, in utility terms, due to the increased riskiness of their consumption profile relative to the perfect risk sharing benchmark, consistent with the findings of Hall and Woodward (2007, 2010). As network VCs and entrepreneurs have to be exposed to idiosyncratic risk to take advantage of their venture capital opportunities — which might be due to moral hazard frictions and the corresponding need for high-powered incentives — they cannot deviate to a less risky consumption profile when pursuing these opportunities.

While imperfect risk sharing thus strongly affects agents' utility from performance-sensitive VC contracts, it also materially affects the value agents assign to the impact of *aggregate* VC investment. I repeat the Lucas (1987) type calculation performed in Section 4.2.1 (see last paragraph of Appendix A.6 for details). I find that the fraction of lifetime consumption that agents would be willing to give up to avoid losing VC investments' contribution to their consumption growth drops by 8 percent relative to the fraction found

⁵⁷Consistent with the calibration in Section 4.1 the parameters $\delta(Z)$ are chosen such that aggregate consumption growth still matches the values in Chen (2010) state by state.

under perfect risk sharing.⁵⁸ While this is a significant reduction, there remain substantial benefits from aggregate VC activity for at least three reasons. First, a substantial part of idiosyncratic venture risk is held by VC investors that can diversify these risks. Second, not all compensation in venture capital is performance-sensitive and exposed to idiosyncratic risk.⁵⁹ Third, VC-backed innovations create externalities that have a positive impact on technological progress and aggregate growth, benefiting agents without exposing them to large idiosyncratic risk (see also the discussion at the end of Section 4.2.1).

Overall, the analysis suggests that whereas quantities and prices of assets traded in competitive financial markets remain similar, agents' utility from performance-sensitive VC compensation and aggregate VC activity is significantly affected by imperfect risk sharing.

6. Conclusion

I present a model that can explain central empirical facts about the magnitude and cyclicity of VC activity. The framework reveals that boom-bust cycle dynamics and uninsurable idiosyncratic risks associated with VC contracts lead to substantial discounts in the value agents assign to VC investments' growth contributions. Despite these risk discounts, I find that agents' valuations of VC funded innovations significantly exceed actual levels of aggregate commitments to the VC industry. When compared to estimates of the cost of business cycles (see, e.g., Lucas, 2003), the results suggest that regulatory changes, such as those that appear to have led to the emergence of the VC industry, can have significant implications for the macroeconomy. The proposed dynamic general equilibrium model is highly tractable, even in the presence of imperfect risk sharing, and could be used as a stepping stone to analyses of policies governing VC activity, entrepreneurship, and investment mandates for institutional investors. Such endeavors are left for future research.

⁵⁸This calculation does not evaluate an agent's loss from moving from an economy with perfect risk sharing to one with imperfect risk sharing. Instead, starting in both types of economies from an allocation where agents hold the (tradable) market portfolio, the calculation determines agents' willingness to pay, given their consumption profiles in those two types of economies. As shown in the last paragraph of Appendix A.6, in the economy with imperfect risk sharing, agents of all skill types then assign the same value to VC investments, as measured by the fraction of lifetime consumption they are willing to give up.

⁵⁹In the model, VC managers also collect performance-insensitive management fees. In addition, agents that are hired by start-ups to support the implementation of entrepreneurs' venture ideas also obtain white-collar wages. In practice, diversification benefits VC managers obtain from investing in multiple ventures are an additional relevant channel reducing idiosyncratic risk exposures (see Hall and Woodward, 2007).

A. Appendix

A.1. Tables

Table 5
Aggregate Consumption Growth

Panels (a) and (b) report the state-contingent values of the consumption growth drifts and the local risk exposures in Chen (2010). Panel (c) reports the stationary distribution.

(a) Consumption growth drifts: $\mu(Z)$				(b) Local risk exposures: $\sigma(Z)$					
		σ					σ		
		low	med	high			low	med	high
	high	0.041	0.048	0.053		high	0.021	0.027	0.032
μ	med	0.018	0.018	0.019	μ	med	0.021	0.027	0.032
	low	-0.004	-0.011	-0.016		low	0.021	0.027	0.032

(c) Unconditional probability: $\Pr[Z]$				
		σ		
		low	med	high
	high	0.016	0.138	0.015
μ	med	0.059	0.542	0.062
	low	0.016	0.138	0.015

Table 6
Discount Rates and White-collar Wages

Panels (a) through (d) report the risk-free rate, the white-collar wage-to-output ratio, and the jump-risk premium and diffusion-risk premium of a levered claim to aggregate consumption (C_t^χ with $\chi = 2.6$) in each of the nine Markov states.

(a) Risk-free rate: $r_f(Z)$					(b) White-collar wages: $\tilde{w}_W(Z)$				
		σ					σ		
		low	med	high			low	med	high
μ	high	0.037	0.039	0.040	μ	high	0.106	0.108	0.110
	med	0.023	0.021	0.019		med	0.098	0.097	0.097
	low	0.008	0.001	-0.005		low	0.092	0.090	0.088

(c) Jump-risk premium: $rp_\chi^J(Z)$					(d) Diffusion-risk premium: $rp_\chi^D(Z)$				
		σ					σ		
		low	med	high			low	med	high
μ	high	0.020	0.028	0.037	μ	high	0.020	0.034	0.048
	med	0.011	0.016	0.023		med	0.020	0.034	0.048
	low	0.020	0.027	0.040		low	0.020	0.034	0.048

Table 7
State-contingent Outcomes in Industries Ψ''

Panel (a) reports the state-contingent values of productivity in industries Ψ'' , which follow from the calibrated relation:

$$\theta(v, Z) = a + b \cdot (\mu(Z) - \min_{\forall Z} \{\mu(Z)\}), \quad (32)$$

where, $a = 0.062$, $b = 0.11$, and where $\mu(Z) \forall Z$ are the consumption growth drifts from Chen (2010). Panel (b) reports the hazard rate of entry \bar{h} scaled by productivity θ in industries Ψ'' .

(a) Productivity: $\theta(v, Z)$					(b) Entry-to-productivity ratio: $\frac{\bar{h}(v, Z)}{\theta(v, Z)}$				
		σ					σ		
		low	med	high			low	med	high
μ	high	0.068	0.069	0.070	μ	high	0.501	0.495	0.489
	med	0.066	0.066	0.066		med	0.518	0.518	0.518
	low	0.063	0.063	0.062		low	0.525	0.525	0.525

A.2. Definition of Allocation and Decentralized Equilibrium

DEFINITION 1 (Allocation). *An allocation in this economy is given by stochastic processes of agents' consumption $[C_j(t)]_{j \in \Phi, t=0}^\infty$, where $j = (S, i_G, i_S)$, quantities of superior and regular venture ideas undertaken in each industry, $[n_1(v, t), n_2(v, t)]_{v \in \Psi, t=0}^\infty$, acquisition of industry expertise in each industry $[\iota_1(v, t)]_{v \in \Psi, t=0}^\infty$, qualities of leading-edge intermediate goods in each industry $[q(v, t)]_{v \in \Psi, t=0}^\infty$, and quantities of intermediate goods produced in each industry $[x(v, t)]_{v \in \Psi, t=0}^\infty$.*

DEFINITION 2 (Decentralized Equilibrium). *An equilibrium in this economy is given by an allocation, and stochastic processes for wage rates $[w_B(t), w_W(t)]_{t=0}^\infty$, the venture ownership stakes obtained by network VCs and entrepreneurs in each industry, the stochastic discount factor $[\xi(t)]_{t=0}^\infty$, and intermediate goods prices denoted by $[p^x(v, t)]_{v \in \Psi, t=0}^\infty$, such that firms' pricing, production, and venture funding maximizes their discounted value; VC funds' industry expertise acquisition and funding decisions maximize their discounted value; agents choose their paths of consumption optimally; labor markets, intermediate good markets, and the final good market clear.*

A.3. Incumbent Optimization and Proof of Proposition 2

First, I show why a firm does not fund additional venture ideas in a product line where it already owns the best patent. The setup ensures that an incumbent's maximization problem is separable across industries, since each incumbent is restricted to operate in a finite number of industries v , which have zero measure. Let $i \geq 1$ denote the number of innovations that the current incumbent's best patent is ahead relative to the next-best patent owned by a different firm, that is, the firm's patent yields quality advantage of a factor $\kappa(v, t) = \kappa^i$. The value the incumbent firm obtains from this patent is given by:

$$P_i(v, t) = \mathbb{E}_t \left[\int_t^{\tau^*} \frac{\xi_\tau}{\xi_t} Y_\tau \left(1 - \frac{1}{\kappa^i} \right) d\tau + \frac{\xi_{s^*}}{\xi_t} P_{i+1}(v, s^*) \right], \quad (33)$$

where the distributions of $\tau^* \equiv \inf \{ \tau : M(v, \tau) > M(v, t) \}$ and $s^* \equiv \inf \{ s : \kappa(v, s) > \kappa(v, t) \}$ are affected by the incumbents optimal venture funding policy. Suppose the associated value function takes the form:

$$P_i(v, Y_t, Z_t) = Y_t \cdot \tilde{P}(v, Z_t) \frac{1 - \frac{1}{\kappa^i}}{1 - \frac{1}{\kappa}}. \quad (34)$$

Since incumbents and non-incumbent firms have symmetric access to regular venture ideas, the per-venture success rate and the per-venture funding cost are equal for both types of firms. To compare incumbent and non-incumbent firms' incentives to fund regular venture ideas it thus suffices to compare each firm type's gain conditional on a venture success. If an incumbent firm succeeds in innovating again in the same product

line, its value jumps by:

$$\begin{aligned}
\tilde{P}_{i+1}(v, Y, Z) - \tilde{P}_i(v, Y, Z) &= Y \cdot \tilde{P}(v, Z) \frac{1 - \frac{1}{\kappa^{i+1}}}{1 - \frac{1}{\kappa}} - Y \cdot \tilde{P}(v, Z) \frac{1 - \frac{1}{\kappa^i}}{1 - \frac{1}{\kappa}} \\
&= Y \cdot \tilde{P}(v, Z) \frac{1}{\kappa^i} \\
&< Y \cdot \tilde{P}(v, Z),
\end{aligned} \tag{35}$$

where the last inequality holds since $\kappa > 1$. Thus, under the conjectured value function, incumbent firms gain less than non-incumbent firms, which gain $Y \cdot \tilde{P}(v, Z)$ conditional on a venture success. As the free-entry condition (19) implies that non-incumbent firms just break even at the equilibrium per-venture success rate and per-venture funding cost, incumbent firms cannot break even when funding ventures in their own product line, as they gain less from innovating (see inequality (35)). Thus, incumbent firms optimally refrain from funding additional ventures in the product line where they already own the best patent.

Finally, I verify the conjectured value function (34), which also proves Proposition 2. As shown above, under this conjectured value function, incumbents optimally do not fund additional venture ideas in their own product line, and the arrival rate of new innovations \bar{h} in the product line is determined as described in Proposition 1. Substituting the conjectured value function (34) into the HJB equation associated with (33) yields the following linear system of equations determining the values $\tilde{P}(v, Z)$ for all $Z \in \Omega$:

$$\begin{aligned}
0 &= 1 - \frac{1}{\kappa^i} - (r_f(Z) + \gamma\sigma^2(Z) + \bar{h}(v, Z) - \mu(Z)) \tilde{P}(v, Z) \frac{1 - \frac{1}{\kappa^i}}{1 - \frac{1}{\kappa}} \\
&\quad - \sum_{Z' \neq Z} \lambda_{ZZ'} e^{\zeta(Z, Z')} \left(\tilde{P}(v, Z') - \tilde{P}(v, Z) \right) \frac{1 - \frac{1}{\kappa^i}}{1 - \frac{1}{\kappa}},
\end{aligned} \tag{36}$$

confirming that the conjecture (34) is indeed a solution to the firm's HJB equation. Given that at date $t = 0$ every incumbent's patent has a quality advantage of a factor κ , the only relevant case is $\kappa(v, t) = \kappa$. Finally, setting $i = 1$ in equation (36), rearranging, and defining the diffusion- and jump-risk premia

$$rp^D(v, Z) = \gamma\sigma^2(Z), \quad rp^J(v, Z) = \sum_{Z' \neq Z} \lambda_{ZZ'} \left(e^{\zeta(Z, Z')} - 1 \right) \left(1 - \frac{\tilde{P}(v, Z')}{\tilde{P}(v, Z)} \right), \tag{37}$$

yields equation (23) in Proposition (2).

A.4. Proof of Proposition 3

In an economy where incumbent firms in each industry $v \in \Psi$ have a quality advantage of a factor κ relative to the next-best producer, incumbents' profit maximization in combination with the market clearing condition for blue-collar labor (27) yield a symmetric allocation of blue-collar labor across industries, that is, $x(v, t) = 1$ for all $v \in \Psi$. Plugging this result into the final good production technology yields:

$$Y_t = A_t Q_t, \tag{38}$$

where I define:

$$Q_t \equiv \exp \left(\int_{\Psi} \log [q(v, t)] dv \right). \quad (39)$$

Innovation arrivals in each industry v follow idiosyncratic Poisson processes with hazard rates $\bar{h}(v, Z)$ defined in equation (21). The aggregate quality level Q_t then grows at the state-dependent rate

$$\frac{dQ_t}{Q_t} = \log [\kappa] \int_{\Psi} \bar{h}(v, Z_t) dv. \quad (40)$$

Thus, final good output follows the stochastic differential equation

$$\frac{dY_t}{Y_t} = \left(\delta(Z_t) + \log [\kappa] \int_{\Psi} \bar{h}(v, Z_t) dv \right) dt + \sigma(Z_t) dB_t. \quad (41)$$

A.5. Proof of Proposition 4

In the baseline analysis of the model, agents can optimally share risks by trading state-contingent claims to future compensation from labor contracts, in particular, white-collar and blue-collar wages, and venture ownership shares provided to network VCs and entrepreneurs. As a result of perfect risk sharing, agents consume constant fractions of aggregate output. If at date 0 agent $j = (i_G, S, i_S)$ has a wealth share ϖ_j , then perfect risk sharing implies that this agent consumes at any time $t \geq 0$:

$$C_j(t) = \varpi_j Y_t. \quad (42)$$

The agent's value function is given by

$$J(\varpi_j, Y_t, Z_t) = \mathbb{E}_t \left[\int_t^{\infty} m(\varpi_j Y_{\tau}, J_{\tau}) d\tau \right], \quad (43)$$

which yields the associated Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} 0 &= m(\varpi_j Y, J(\varpi_j, Y, Z)) + J_Y(\varpi_j, Y, Z) Y \mu(Z) + \frac{1}{2} J_{YY}(\varpi_j, Y, Z) Y^2 \sigma^2(Z) \\ &+ \sum_{Z' \neq Z} \lambda_{ZZ'} (J(\varpi_j, Y, Z') - J(\varpi_j, Y, Z)). \end{aligned} \quad (44)$$

Due to the isoelastic properties of the setup, I conjecture the solution for J takes the form

$$J(\varpi_j, Y, Z) = F(Z) \frac{Y^{1-\gamma}}{1-\gamma} \varpi_j^{1-\gamma}. \quad (45)$$

Substituting the conjecture (45) into the HJB equation yields the following system of equations that $F(Z)$ solves for all $Z \in \Omega$:

$$0 = \left(\frac{\beta(1-\gamma)}{\rho} \left(F(Z)^{-\frac{\rho}{1-\gamma}} - 1 \right) + (1-\gamma)\mu(Z) - \frac{1}{2}\gamma(1-\gamma)\sigma^2(Z) \right) F(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (F(Z') - F(Z)). \quad (46)$$

Duffie and Epstein (1992b) show that household maximization implies that a state-pricing process ξ_t may be written as follows:

$$\xi_t \equiv \exp \left[\int_0^t m_J(C_j(\tau), J_\tau) d\tau \right] m_C(C_j(t), J_t). \quad (47)$$

Using the value function (45) and the consumption policy (42) we obtain

$$\xi_t = (\varpi_j Y_t)^{-\gamma} \beta F(Z_t)^{1-\frac{\rho}{1-\gamma}} e^{\left\{ \int_0^t \left(\frac{\beta(1-\gamma-\rho)}{\rho} F(Z_\tau)^{-\frac{\rho}{1-\gamma}} - \frac{\beta(1-\gamma)}{\rho} \right) d\tau \right\}}. \quad (48)$$

Applying Itô's lemma yields

$$\frac{d\xi_t}{\xi_{t-}} = -r_f(Z_t) dt - \gamma\sigma(Z_t) dB_t + \sum_{Z' \neq Z_{t-}} \left(\left(\frac{F(Z')}{F(Z_{t-})} \right)^{1-\frac{\rho}{1-\gamma}} - 1 \right) (dN_t(Z_{t-}, Z') - \lambda_{Z_{t-}, Z'} dt), \quad (49)$$

where

$$r_f(Z) = -\frac{\mathbb{E}_t \frac{d\xi_t}{\xi_{t-}}}{\xi_{t-}} = \frac{\beta(1-\gamma)}{\rho} - \frac{\beta(1-\gamma-\rho)}{\rho} F(Z)^{-\frac{\rho}{1-\gamma}} + \gamma\mu(Z) - \frac{1}{2}\gamma(1+\gamma)\sigma^2(Z) - \sum_{Z' \neq Z} \lambda_{ZZ'} \left(\left(\frac{F(Z')}{F(Z)} \right)^{1-\frac{\rho}{1-\gamma}} - 1 \right). \quad (50)$$

Note that the growth of ξ_t is independent of an agent's wealth share ϖ_j . Thus, all agents in the economy assign identical values to payoffs in all future states of the world, and have no incentives to deviate from the consumption policy (42). Defining

$$\zeta(Z, Z') \equiv \left(1 - \frac{\rho}{1-\gamma} \right) \log \left(\frac{F(Z')}{F(Z)} \right). \quad (51)$$

and rearranging (49) yields the result stated in Proposition 4.

A.6. Imperfect Risk Sharing

For the analysis of the model discussed in Section 5 the contractual space and the logical order of events within a period $[t, t + dt]$ are specified as follows:

1. Agents complete all financial transactions and choose their consumption for the period. Financial transactions can specify payments conditional on the paths of the variables $\{Y_\tau\}_{\tau \geq t}$, $\{Z_\tau\}_{\tau \geq t}$,

$\{\iota_1(v, \tau)\}_{v \in \Psi, \tau \geq t}$, $\{M(v, \tau)\}_{v \in \Psi, \tau \geq t}$, and $\{M_1(v, \tau)\}_{v \in \Psi, \tau \geq t}$, where $M_1(v, t)$ is a counting process keeping track of the number of new patents created from funding the most productive entrepreneurs' venture ideas in industry v . VC funds and firms can obtain funding commitments from investors as a function of these variables. In return, investors obtain venture ownership shares.

2. The random variables $u_{V,t}$ and $u_{E,t}$ are realized, that is, nature chooses the network VCs with connections and the most productive entrepreneurs in the period.
3. Whenever active VC intermediation obtains ($\iota_1(v, t) = 1$) the connected network VC and the most productive entrepreneur have to be compensated with venture equity, as specified in the baseline model. Network VCs that obtained connections choose whether to facilitate industry expertise acquisition, which determines the realizations of the variables $\{\iota_1(v, t)\}_{v \in \Psi}$.
4. State-contingent funding commitments for VC funds are fulfilled (contingent on $\iota_1(v, t) = 1$) and are used to pay for the implementation of venture ideas of the most productive entrepreneurs and for VC management fees.
5. State-contingent funding commitments for regular venture ideas are fulfilled and can be used to pay for investment cost.
6. Innovation arrivals as reflected by jumps in the variables $\{M(v, t)\}_{v \in \Psi}$ are realized and patents are allocated to ventures as described in the baseline setup.

For the analysis of the economy with incomplete risk sharing it is useful to first consider a setting where, in each group i_G , there is a discrete number ($K+1$) of agents in each skill subgroup S (instead of a continuum of agents). Define $\Delta i_S \equiv \frac{1}{K+1}$ and let $i_S \in \{0, \frac{1}{K}, \frac{2}{K}, \dots, 1\}$ again index agents of skill $S \in \{E, V, W, B\}$. To establish the solution for the model with continuums, I take the limit $K \rightarrow \infty$, or equivalently, $\Delta i_S \downarrow 0$.

Probabilities of receiving venture opportunities. The matching process described in Sections 2.4 and 2.5 implies that, in each period $[t, t + dt]$, each group i_G has one network VC with connections and one entrepreneur with the most productive ideas in some industry $v \in \Psi$. Network VCs within a group have a uniform propensity of receiving this connection. Similarly, the entrepreneurs in a group have a uniform propensity of becoming the most productive entrepreneur. In the setting with a discrete number of agents per group, each entrepreneur and each network VC has a probability Δi_S of receiving a VC-related opportunity (that is, more productive ideas or connections) in each period. Such an agent's probability of receiving an opportunity in a specific industry v is thus $\Delta i_S dv$.

Consumption and portfolio policies. I start with the conjecture that, in equilibrium, all agents rebalance their portfolios at the beginning of each period to always invest all their tradable wealth in the "market portfolio" of all tradable assets, the price of which is denoted by P_T . For example, immediately after experiencing a positive idiosyncratic venture success as an entrepreneur, an agent sells her equity stake to other agents at the market price P and rebalances to again hold the market portfolio. Further, conjecture that all agents choose the same consumption-to-tradable-wealth ratios, which implies that the wealth shares also pin down the fraction of output Y that each agent consumes. Below I will verify the optimality of these conjectured consumption and portfolio policies.

Let $\varpi(S, i_G, i_S, t)$ denote the share of total tradable wealth owned by agent (S, i_G, i_S) at time t scaled by $\frac{4}{di_G \Delta i_S}$, such that the actual wealth share is: $\varpi(S, i_G, i_S, t) \frac{di_G \Delta i_S}{4}$. By definition, wealth shares add up

to one:

$$\sum_{S \in \{B, W, V, E\}} \sum_{i_S \in \{0, \frac{1}{K}, \frac{2}{K}, \dots, 1\}} \int_0^1 \varpi(S, i_G, i_S, t) \frac{di_G \Delta i_S}{4} = 1. \quad (52)$$

To economize on notation, I will use a subscript j to refer to an agent identified by a tuple (S, i_G, i_S) going forward. An agent's tradable wealth at time t is denoted by $W_j(t)$ and is given by:

$$W_j(t) = P_T(t) \varpi_j(t) \frac{di_G \Delta i_S}{4}. \quad (53)$$

The assets that are part of the tradable market portfolio pay an aggregate dividend that is equal to the aggregate output Y . Thus, an agent holding a fraction $\varpi_j(t) \frac{di_G \Delta i_S}{4}$ of the market portfolio obtains a dividend $Y_t \varpi_j(t) \frac{di_G \Delta i_S}{4}$. Identical consumption-to-wealth ratios and market clearing imply that agents consume this dividend, that is,

$$C_j(t) = Y_t \varpi_j(t) \frac{di_G \Delta i_S}{4}. \quad (54)$$

Evolution of tradable wealth shares. The set of assets in positive net supply that are tradable changes over time. The venture ownership stakes network VCs and entrepreneurs will obtain through future idiosyncratic shocks are not traded until after ventures succeed and ownership stakes can be sold in the competitive market. It is useful to specify the market values of different vintages of tradable market portfolios. I define $P_{T, \tau}(t)$ as the time- t market value of the portfolio of all assets in positive net supply that have been tradable since time τ , where $t \geq \tau$.

Let $\vartheta_j(v, t) \in [0, 1]$ denote the equity stake agent j obtains *if* she receives a VC-related opportunity in industry v in period $[t, t + dt]$, which is relevant only for entrepreneurs and network VCs (that is, $\vartheta_j(v, t) = 0$ for white-collar and blue-collar agents). Further let $\mathbf{1}_j(v, t)$ be an indicator variable that takes the value one if agent j actually obtains a VC-related opportunity in industry v in period $[t, t + dt]$. An agent obtains equity worth $\vartheta_j(v, t) P(v, t) dv$ when succeeding with a VC-backed venture in industry v . Given the conjectured consumption and portfolio policies an agent's tradable wealth thus evolves as follows:

$$dW_j(t) = \varpi_j(t) dP_{T, t}(t) \frac{di_G \Delta i_S}{4} + \int_{\Psi} dM_1(v, t) \mathbf{1}_j(v, t) \vartheta_j(v, t) P(v, t) dv. \quad (55)$$

The total fraction of venture equity that entrepreneurs and network VCs will obtain jointly in industry v at time t is given by

$$\vartheta(v, t) = \begin{cases} \frac{\pi_1(v, t)}{h_1(v, t) P(v, t)} & \text{if } h_1(v, t) > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (56)$$

These equity shares are distributed according to the bargaining power parameter ϱ . The remainder share of the equity is held by diversified investors and is part of the market portfolio of tradable assets.

The dynamics of the value of the market portfolio of tradable assets can be decomposed into the value change of existing tradable assets and the arrival of new venture equity sold by successful entrepreneurs and

network VCs:

$$\begin{aligned}
dP_T(t) &= dP_{T,t}(t) + \int_{\Psi} \vartheta(v, t) dM_1(v, t) P(v, t) dv \\
&= dP_{T,t}(t) + \int_{\Psi} \vartheta(v, t) h_1(v, t) P(v, t) dv dt \\
&= dP_{T,t}(t) + \int_{\Psi} \pi_1(v, t) dv dt,
\end{aligned} \tag{57}$$

where the first step obtains since $\mathbb{E}[dM_1(v, t)] = h_1(v, t) dt$, and the law of large numbers applies (Uhlig, 1996),⁶⁰ and the second step obtains due to the definition of $\vartheta(v, t)$ provided in (56) and since $\pi_1(v, t) = 0$ when $h_1(v, t) = 0$. Thus, we obtain the following law of motion for $\varpi_j(t)$:

$$\begin{aligned}
d\varpi_j(t) &= d\left(\frac{W_j(t)}{P_T(t)^{\frac{di_G \Delta i_S}{4}}}\right) \\
&= \frac{dW_j(t)}{P_T(t)^{\frac{di_G \Delta i_S}{4}}} - \frac{W_j(t)}{P_T(t)^{\frac{di_G \Delta i_S}{4}}} \frac{dP_T(t)}{P_T(t)} \\
&= \frac{\varpi_j(t) dP_{T,t}(t)^{\frac{di_G \Delta i_S}{4}} + \int_{\Psi} dM_1(v, t) \mathbf{1}_j(v, t) \vartheta_j(v, t) P(v, t) dv}{P_T(t)^{\frac{di_G \Delta i_S}{4}}} - \varpi_j(t) \frac{dP_{T,t}(t) + \int_{\Psi} \pi_1(v, t) dv dt}{P_T(t)} \\
&= \frac{\int_{\Psi} dM_1(v, t) \mathbf{1}_j(v, t) \vartheta_j(v, t) P(v, t) dv}{P_T(t)^{\frac{\Delta i_S di_G}{4}}} - \varpi_j(t) \frac{\int_{\Psi} \pi_1(v, t) dv}{P_T(t)} dt,
\end{aligned} \tag{58}$$

which yields:

$$\frac{d\varpi_j(t)}{\varpi_j(t)} = \frac{\int_{\Psi} dM_1(v, t) \mathbf{1}_j(v, t) \vartheta_j(v, t) P(v, t) dv}{\varpi_j(t) P_T(t)^{\frac{\Delta i_S di_G}{4}}} - \frac{\int_{\Psi} \pi_1(v, t) dv}{P_T(t)} dt. \tag{59}$$

The first term reflects idiosyncratic risk — each agent j obtains a venture equity share $\vartheta_j(v, t) > 0$ in at most one industry v and thus, may be exposed to venture success risk in that particular industry. The second term reflects the decline in an agent's wealth share due to the venture successes of *other* agents (network VCs and entrepreneurs) in the economy.

Consumption dynamics. Given the consumption policy (54), agent j 's consumption follows:

$$\frac{dC_j(t)}{C_j(t)} = \left(\mu(t) - \frac{\int_{\Psi} \pi_1(v, t) dv}{P_T(t)}\right) dt + \sigma(t) dB_t + \frac{\int_{\Psi} dM_1(v, t) \mathbf{1}_j(v, t) \vartheta_j(v, t) P(v, t) dv}{\varpi_j(t) P_T(t)^{\frac{\Delta i_S di_G}{4}}}. \tag{60}$$

Value function. Under the conjectured consumption policy an agent's value function is given by:

$$J(\varpi_j(t), Y_t, Z_t) = \mathbb{E}_t \left[\int_t^{\infty} m(C_j(\tau), J_{\tau}) d\tau \right], \tag{61}$$

⁶⁰In the calibration, $dM_1(v, t)$, with $v \in \Psi'$ is a collection of identically distributed and pairwise uncorrelated random variables with common finite mean and variance. In industries $v \in \Psi''$ VC funds are not active, such that $h_1(v, t) = 0$ and $dM_1(v, t) = 0$ at all times.

which yields the associated HJB equation:

$$\begin{aligned}
0 = & m(C_j, J(\varpi_j, Y, Z)) + J_Y(\varpi_j, Y, Z) Y \mu(Z) - J_{\varpi_j}(\varpi_j, Y, Z) \varpi_j \frac{\int_{\Psi} \tilde{\pi}_1(v, Z) dv}{\tilde{P}_T(Z)} \\
& + \frac{1}{2} J_{YY}(\varpi_j, Y, Z) Y^2 \sigma^2(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (J(\varpi_j, Y, Z') - J(\varpi_j, Y, Z)) \\
& + \int_{\Psi} \left(J \left(\varpi_j \cdot \left(1 + \frac{\vartheta_j(v, Z) \tilde{P}(v, Z) dv}{\varpi_j \tilde{P}_T(Z) \frac{\Delta i_S di_G}{4}} \right), Y, Z \right) - J(\varpi_j, Y, Z) \right) \bar{l}_1(v, Z) h_1(v, Z) \Delta i_S dv, \quad (62)
\end{aligned}$$

where I use the fact $\mathbb{E}[dM_1(v, t)|Z] = \bar{l}_1(v, Z) h_1(v, Z)$, and that $\Pr[\mathbf{1}_j(v, t) = 1] = \Delta i_S dv$, if agent j is an entrepreneur or a network VC. Note that for white-collar and blue-collar agents we have $\vartheta_j(v, Z) = 0$, such that the last term in the previous equation drops out. Conjecture the solution for J takes the form:

$$J(\varpi_j, Y, Z) = F(Z) \frac{(\varpi_j Y)^{1-\gamma}}{1-\gamma}. \quad (63)$$

Substituting this conjecture into the HJB equation yields:

$$\begin{aligned}
0 = & \left(\frac{\beta(1-\gamma)}{\rho} \left(F(Z)^{-\frac{\rho}{1-\gamma}} - 1 \right) + (1-\gamma) \left(\mu(Z) - \frac{\int_0^1 \tilde{\pi}_1(v, Z) dv}{\tilde{P}_T(Z)} \right) - \frac{1}{2} \gamma(1-\gamma) \sigma^2(Z) \right) F(Z) \\
& + \sum_{Z' \neq Z} \lambda_{ZZ'} (F(Z') - F(Z)) + \int_{\Psi} \Delta i_S \left(\left(1 + \frac{\vartheta_j(v, Z) \tilde{P}(v, Z_t) dv}{\varpi_j \tilde{P}_T(Z_t) \frac{\Delta i_S di_G}{4}} \right)^{1-\gamma} - 1 \right) F(Z) \bar{l}_1(v, Z) h_1(v, Z) dv. \quad (64)
\end{aligned}$$

For $\gamma > 1$, the value of the term

$$\left(\left(1 + \frac{\vartheta_j(v, Z) \tilde{P}(v, Z_t) dv}{\varpi_j \tilde{P}_T(Z_t) \frac{\Delta i_S di_G}{4}} \right)^{1-\gamma} - 1 \right) \quad (65)$$

is bounded from below by -1 and from above by 0 . Thus, we obtain in the relevant limiting case:

$$\lim_{\Delta i_S \downarrow 0} \left\{ \Delta i_S \cdot \left(\left(1 + \frac{\vartheta_j(v, Z) \tilde{P}(v, Z_t) dv}{\varpi_j \tilde{P}_T(Z_t) \frac{\Delta i_S di_G}{4}} \right)^{1-\gamma} - 1 \right) F(Z) \bar{l}_1(v, Z) h_1(v, Z) \right\} = 0. \quad (66)$$

It follows that this term drops from equation (64) in the relevant case when $\lim_{\Delta i_S \downarrow 0}$, and $F(Z)$ solves:

$$\begin{aligned}
0 = & \left(\frac{\beta(1-\gamma)}{\rho} \left(F(Z)^{-\frac{\rho}{1-\gamma}} - 1 \right) + (1-\gamma) \left(\mu(Z) - \frac{\int_{\Psi} \tilde{\pi}_1(v, Z) dv}{\tilde{P}_T(Z)} \right) \right. \\
& \left. - \frac{1}{2} \gamma(1-\gamma) \sigma^2(Z) \right) F(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (F(Z') - F(Z)). \quad (67)
\end{aligned}$$

Note that this equation is independent of an agent's skill type S and her wealth share ϖ_j , implying that all agents share the same $F(Z)$ function, consistent with the conjecture of the value function.

Verifying the conjectured equilibrium for $\lim_{\Delta i_S \downarrow 0}$. I verify whether any individual agent has an incentive to deviate from the posited behavior, given that all other agents behave as conjectured. The

marginal utility process of agents following the conjectured portfolio and consumption policies may be written as follows:

$$\xi_t \equiv \exp \left[\int_0^t m_J(C_j(\tau), J_\tau) d\tau \right] m_C(C_j(t), J_t). \quad (68)$$

Using the value function (63) associated with the conjectured policies, where F solves equation (67), we obtain

$$\xi_t = (\varpi_j Y_t)^{-\gamma} \beta F(Z_t)^{1-\frac{\rho}{1-\gamma}} e^{\left\{ \int_0^t \left(\frac{\beta(1-\gamma-\rho)}{\rho} F(Z_\tau)^{-\frac{\rho}{1-\gamma}} - \frac{\beta(1-\gamma)}{\rho} \right) d\tau \right\}}. \quad (69)$$

Applying Itô's lemma to obtain $r_f(Z) = -\frac{\mathbb{E}_t \frac{d\xi_t}{\xi_t}}{\xi_t}$ yields:

$$\begin{aligned} r_f(Z) &= \frac{\beta(1-\gamma)}{\rho} - \frac{\beta(1-\gamma-\rho)}{\rho} F(Z)^{-\frac{\rho}{1-\gamma}} + \gamma \left(\mu(Z) - \frac{\int_{\Psi} \tilde{\pi}_1(v, Z) dv}{\tilde{P}_T(Z)} \right) - \frac{1}{2} \gamma(1+\gamma) \sigma^2(Z) \\ &\quad - \sum_{Z' \neq Z} \lambda_{ZZ'} \left(\left(\frac{F(Z')}{F(Z)} \right)^{1-\frac{\rho}{1-\gamma}} - 1 \right) \\ &\quad - \int_{\Psi} \Delta i_S \left(\left(1 + \frac{\vartheta_j(v, Z) \tilde{P}(v, Z_t) dv}{\varpi_j \tilde{P}_T(Z_t) \frac{\Delta i_S di_G}{4}} \right)^{-\gamma} - 1 \right) \bar{l}_1(v, Z) h_1(v, Z) dv. \end{aligned} \quad (70)$$

where, for the relevant limiting case $\Delta i_S \downarrow 0$, the last term drops out, since:

$$\lim_{\Delta i_S \downarrow 0} \left\{ \Delta i_S \cdot \left(\left(1 + \frac{\vartheta_j(v, Z) \tilde{P}(v, Z_t) dv}{\varpi_j \tilde{P}_T(Z_t) \frac{\Delta i_S di_G}{4}} \right)^{-\gamma} - 1 \right) \bar{l}_1(v, Z) h_1(v, Z) \right\} = 0. \quad (71)$$

It follows that all agents agree on the risk-free rate $r_f(Z)$ independently of their skill S and ϖ_j . The dynamics for ξ_t are given by:

$$\begin{aligned} \frac{d\xi_t}{\xi_{t-}} &= -r_f(Z_t) dt - \gamma \sigma(Z_t) dB_t + \sum_{Z' \neq Z_{t-}} \left(\left(\frac{F(Z')}{F(Z_{t-})} \right)^{1-\frac{\rho}{1-\gamma}} - 1 \right) (dN_t(Z_{t-}, Z') - \lambda_{Z_{t-} Z'} dt) \\ &\quad + \int_{\Psi} \mathbf{1}_j(v, t) \left(\left(1 + \frac{\vartheta_j(v, Z) \tilde{P}(v, Z_t) dv}{\varpi_j(t) \tilde{P}_T(Z_t) \frac{\Delta i_S di_G}{4}} \right)^{-\gamma} - 1 \right) dM_1(v, t). \end{aligned} \quad (72)$$

A central feature of the analysis with imperfect risk sharing is that agents cannot trade claims that pay conditional on the joint future realizations of their idiosyncratic venture opportunities ($\mathbf{1}_j(v, t)$ and $\vartheta_j(v, t)$) and VC-backed venture successes ($dM_1(v, t)$). Yet consider the set of tradable claims, which can pay a cash flow CF at some future date $\tau > t$ contingent on realizations of the aggregate state variables Y and Z , the quality ladder counting processes $\{M(v, \tau)\}_{v \in \Psi}$, the VC-backed patent counting processes $\{M_1(v, \tau)\}_{v \in \Psi}$, and the VC intermediation variables $\{\iota_1(v, \tau)\}_{v \in \Psi}$. Formally, any tradable claim to a cash flow is specified as a general function of these state variables:

$$CF_\tau = CF(Y_\tau, Z_\tau, \{\iota_1(v, \tau)\}_{v \in \Psi}, \{M(v, \tau)\}_{v \in [0,1]}, \{M_1(v, \tau)\}_{v \in [0,1]}) \quad (73)$$

Define “ ns ” (no success) as the event where

$$\int_t^\tau \int_{\Psi} \mathbf{1}_j(v, s) \vartheta_j(v, Z) dM_1(v, s) ds = 0, \quad (74)$$

that is, agent j does not obtain idiosyncratic venture equity gains between time t and time τ . Define $PNS_j(t, \tau, Z)$, as the probability of this event, which will generally depend on the current state Z and the type of the agent j . $PNS_j(t, \tau, Z)$ solves for all Z :

$$0 = \frac{\partial PNS_j(t, \tau, Z)}{\partial t} - \frac{1}{dt} \mathbb{E} \left[\int_{\Psi} \mathbf{1}_j(v, s) \vartheta_j(v, Z) dM_1(v, t) \right] PNS_j(t, \tau, Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (PNS_j(t, \tau, Z') - PNS_j(t, \tau, Z)) \quad (75)$$

with $PNS_j(\tau, \tau, Z) = 1$. Note that in the relevant case where $\Delta i_S \downarrow 0$ we obtain:

$$\begin{aligned} & \lim_{\Delta i_S \downarrow 0} \left\{ \frac{1}{dt} \mathbb{E} \left[\int_{\Psi} \mathbf{1}_j(v, s) \vartheta_j(v, Z) dM_1(v, t) \right] \right\} \\ &= \lim_{\Delta i_S \downarrow 0} \left\{ \Delta i_S \int_{\Psi} \vartheta_j(v, Z) \bar{t}_1(v, Z) h_1(v, Z) dv \right\} \\ &= 0 \end{aligned} \quad (76)$$

It follows that $PNS_j(t, \tau, Z) = 1$. Agents then value a claim to CF_τ as follows:

$$\begin{aligned} \mathbb{E}_t \left[\frac{\xi_\tau}{\xi_t} CF_\tau \right] &= \mathbb{E}_t \left[\frac{\xi_\tau}{\xi_t} CF_\tau \middle| ns \right] PNS_j(t, \tau, Z_t) + \mathbb{E}_t \left[\frac{\xi_\tau}{\xi_t} CF_\tau \middle| ns^c \right] (1 - PNS_j(t, \tau, Z_t)) \\ &= \mathbb{E}_t \left[\frac{\xi_\tau}{\xi_t} CF_\tau \middle| ns \right], \end{aligned} \quad (77)$$

that is, the tradable claim is valued by all agents *as if* the evolution of their ξ_t was given by:

$$\begin{aligned} \frac{d\xi_t}{\xi_{t-}} &= -r_f(Z_t) dt - \gamma \sigma(Z_t) dB_t \\ &+ \sum_{Z' \neq Z_{t-}} \left(\left(\frac{F(Z')}{F(Z_{t-})} \right)^{1-\frac{\rho}{1-\gamma}} - 1 \right) (dN_t(Z_{t-}, Z') - \lambda_{Z_{t-}Z'} dt). \end{aligned} \quad (78)$$

The prices agents assign to tradable claims are thus independent of their own wealth share ϖ_j and their skill S . Since all agents agree on the prices of tradable claims they have no incentive to deviate from the conjectured portfolio- and consumption policies. Thus, the overall wealth distribution does not constitute a state variable, and the only relevant aggregate state variables are Z and Y .

Valuing growth contributions of aggregate VC investment. We have proven that for any agent with a current wealth share $\varpi_j(t)$ the solution for the value function J is given by (63), where $F(Z)$ solves (67). Under the alternative scenario where starting from date t , agents do not expect any future innovations from VC investments, but future state-contingent resource allocations remain unchanged, all agents still have the same tradable wealth shares, as all agents hold the market portfolio, and thus, their tradable wealth is equally affected, in percentage terms, by this change in expectations. The current level of output Y_t is also

unaffected by this change in expectations. However, the function $F(Z)$ now solves:

$$0 = \left(\frac{\beta(1-\gamma)}{\rho} \left(F(Z)^{-\frac{\rho}{1-\gamma}} - 1 \right) + (1-\gamma) \left(\mu(Z) - \log[\kappa] \int_{\Psi} \bar{v}_1(v, Z) h_1(v, Z) dv \right) - \frac{1}{2} \gamma (1-\gamma) \sigma^2(Z) \right) F(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (F(Z') - F(Z)), \quad (79)$$

which accounts for the lower growth expectations. Overall, each agent's value function is equally affected, in percentage terms, by the change in expectations, since each agent experiences an identical change in the $F(Z)$ function. Since agents have identical risk aversion γ , they are also willing to give up the same fraction of lifetime consumption to avoid losing the growth from aggregate VC investments.

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