Delegated Learning in Asset Management*

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ABSTRACT

We develop a tractable framework of delegated asset management with flexible information acquisition in a multi-asset economy in which fund managers face moral hazard in portfolio allocation decisions. We explore the features of the optimal affine compensation contract for fund managers, in which benchmarking arises endogenously as part of their optimal compensation. In the equilibrium with delegated learning, asset prices reflect both the acquired private information of fund managers and their desire to hedge their exposure to the benchmark. We show a potential gap between our model-implied measure and several widely-adopted empirical statistics intended to capture managerial ability. In a multi-period extension, we propose a new performance measure of fund manager skill. Our delegated learning channel can also help rationalize the excess comovement documented in asset returns.

Keywords: Managerial Compensation, Endogenous Benchmark, Moral Hazard, Information Acquisition

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1 Introduction

There is growing concern that active fund managers lack the superior ability in garnering higher returns to justify their higher fees compared to their passive counterparts. Consistent with this view, in recent years there has been an accelerating shift in fund flows from active to passive strategies.[1] The existing literature has focused on either improving empirical measures to evaluate the unobservable skill of fund managers, or on developing theories to justify the lack of empirical support for their superior ability.[2] Despite the progress of this fast growing literature, the relationship between fund manager ability and the incentives that they face, in equilibrium, is still not well-understood. In this paper, we ask to what extent such unobservable ability is an outcome of the incentives provided to active managers to acquire information through their compensation contracts.

To investigate this issue, we cast the information acquisition and portfolio allocation decisions of a delegated asset manager as a principal-agent problem between the fund manager and its investors. We refer to this as the delegated learning channel. We study an economy in which asset managers can trade on behalf of investors in a multi-asset financial market, similar to that in Admati (1985). In the spirit of Kacperczyk et al. (2016), fund managers are able to exert costly effort to learn about the aggregate and asset-specific components of the payoffs of the assets in which they can invest. The inability of investors to observe the effort and portfolio decisions of their delegated asset managers, however, forces investors to offer a contract that is incentive compatible to managers, who seek only to maximize their compensation. In equilibrium, these incentives feed into asset prices as fund managers trade

[1]Since 2005, actively managed equity and fixed income funds have lost fund flows to passive strategies globally. According to MorningStar, over last decade, actively run U.S. stock funds saw net outflows every year, totaling about $600 billion, while their indexed counterparts saw net inflows of approximately $700 billion. See http://www.marquetteassociates.com/research/a-continued-shift-from-active-to-passive-in-u-s-equities.

[2]The existing literature has developed several theories to help explain the lack of empirical support that active managers have superior ability, including that fund performance exhibits decreasing-returns-to-scale (Berk and Green, 2004), that managers choose investments based on their benchmark and flow-performance sensitivity (Brennan (1993), Admati and Pfleiderer (1997), Buffa et al. (2014)), and that skill reflects a choice to acquire information over the business cycle (Kacperczyk et al. (2014), Kacperczyk et al. (2016)).

on their private information in financial markets, which then feed back into the determinants of the optimal contract in the principal-agent problem between the manager and investors.

The optimal affine contract for fund managers that we derive features three components: a fixed fee, a performance-based reward that evaluates a fund manager for its performance, and benchmarking relative to the ex-ante mean-variance efficient portfolio. In contrast to such frameworks as those of Basak and Pavlova (2013) and Buffa et al. (2014), the optimal benchmark we derive in our economy is endogenous and arises as an outcome of the compensation contract. Since the performance-based reward influences the aggressiveness with which fund managers trade on their private information, it feeds into the informativeness of asset prices in equilibrium. Through this channel, the performance-based piece impacts the overall uncertainty that fund managers face when choosing their portfolio, and consequently their incentives to exert effort to acquire private information. By benchmarking, the investor effectively endows the fund manager with a tilted short position in the benchmark portfolio, which leads it to hedge its benchmark risk with direct investors in financial markets. This tilt, consequently, impacts the level of risk-sharing between the investor and manager. Our analysis therefore highlights a separation between information acquisition and risk-sharing incentives in delegated asset management.

To illustrate how the optimal affine contract varies with the asset environment, we perform comparative statics when managers are more risk-averse by altering the overall risk in the economy and the cross-sectional correlation of asset payoffs. As the overall level of uncertainty about asset payoffs increases, the optimal contract places less emphasis on the performance-based component, and more weight on benchmark-based incentives. This is optimal because the marginal benefit of exerting effort to learn is higher for fund managers, even in the absence of incentives, the higher the level of uncertainty in the economy, and the shift toward benchmarking reflects the increased value fund managers are expected to add over direct investment by the fund's investors. When the correlation of payoffs increases, in contrast, the optimal contract instead puts more weight on the performance-based component, and less on benchmarking. This occurs because the increased correlation both reduces the cross-section of risk in the economy about which fund managers can learn, and makes prices more revealing about the aggregate sources of risk. As such, managers reduce the overall effort that they exert to acquire private information, which motivates the need for stronger performance-based incentives and lessens the benefit of benchmarking for sharing risk.

Our model has novel implications for identifying skill among fund managers. We highlight a gap between our model-implied measure of fund manager skill, the reduction in uncertainty about asset payoffs, and empirical statistics meant to capture asset management ability, such as the active share proposed by Cremers and Petajisto (2009) and the return gap of Kacperczyk et al. (2008). When the overall level of payoff uncertainty increases, fund managers devote more effort to acquiring private information and take more active positions, when compared to the benchmark portfolio, and this is correctly reflected in our theoretical analogues of the two empirical measures. When asset payoffs become more correlated, however, fund managers exert less overall effort to learn, but may appear more active because the optimal benchmark, the ex-ante mean-variance efficient portfolio, takes smaller positions in the risky assets because of the diminished benefits from diversification. Consequently, our analysis cautions in the interpretation of these empirical measures as proxies for managerial ability, and also highlights the importance of endogenizing the benchmark for theoretical predictions.

The interaction between incentives and learning also delivers rich cross-sectional implications on asset returns. The hedging demand of fund managers for the benchmark portfolio, for instance, raises the prices of assets held short in the benchmark portfolio, lowering their risk premium in equilibrium to compensate direct investors for providing liquidity. Our model can also help rationalize the excess comovement in asset returns, documented in Pindyck and Rotemberg (1990) and Barberis et al. (2005). As prices serve as an endogenous mechanism for fund managers to coordinate on which private information to acquire, their correlated decisions are amplified in the payoff variation reflected in prices through their trading.

We then investigate two extensions of our model, one in which trading by managers occurs over multiple periods, and one in which we endow investors with background risk that is correlated with the returns on the assets in the economy. The dynamic extension illustrates that having multiple periods introduces intertemporal incentives for fund managers to acquire private information and, more importantly, can provide investors with a time-series of past fund behavior to improve monitoring. We show that the historical variance of a fund's return gap, downweighted by the dispersion of asset payoffs, provides a consistent measure of average portfolio selection skill, and argue how investors learning about a fund manager's skill through this channel could help explain the nonlinear relationship between performance and fund flows observed empirically. With background risk, we show that managers with skill are forced to internalize this background risk by the appropriate adjustment of the benchmark against which they are evaluated. Finally, we distinguish our mechanism of learning by managers from the literature on learning about managers.

Our work is related to the literature on delegated asset management under asymmetric information. García and Vanden (2009) and Gárleanu and Pedersen (2015) explore the implications for market efficiency of the formation of mutual funds in the presence of costly information acquisition in a single asset setting. García and Vanden (2009) also consider a principal-agent model of delegated asset management, yet they model management fees as a fixed fraction of assets under management and assume managers pay a fixed fee to become informed. Our work focuses on the optimal affine contract between investors and fund managers in a multi-asset principal-agent setting. Kapur and Timmermann (2005) investigate the impact of relative performance contracts on the equity premium and on portfolio herding. Dybvig et al. (2010) and He and Xiong (2013) consider the market-timing benefits of benchmarking in a partial equilibrium setting. Kyle et al. (2011) investigates the incentives to acquire information under delegated asset management for a large informed fund, in the spirit of Kyle (1985), while Glode (2011) and Savov (2014) microfound delegated asset management as a vehicle for investors to hedge their background risk. Huang (2015) studies the market for information brokers in an equilibrium setting with optimal contracting to explain home bias, comovement in asset idiosyncratic volatility, and the possibility of herding and equilibria multiplicity.

This paper is connected to the growing literature on equilibrium asset pricing with flexible information acquisition. Van Nieuwerburgh and Veldkamp (2009, 2010) and Kacperczyk et al. (2016) study the flexible information acquisition problem faced by investors who have limited attention that they can allocate to learning about risky asset payoffs, the latter of which focuses on business cycle implications. Maćkowiak and Wiederholt (2012) investigate the information acquisition decisions of investors who have limited liability, while Huang et al. (2016) models information acquisition as part of a dynamic reputation game between the fund and its investors. In contrast to these studies, we model the information acquisition of managers as being subject to agency issues within an equilibrium framework.

In addition, our work is also related to the literature on manager incentives and benchmarking in the asset management industry. Basak and Pavlova (2013) and Buffa et al. (2014) investigate the asset pricing implications of benchmarking against an exogenous index in a multi-asset setting, with Buffa et al. (2014) embedding benchmarking in a principal-agent framework. Buffa and Hodor (2017) explore the asset pricing implications of heterogeneous benchmarking. Starks (1987) studies the role of symmetric versus bonus performance-based contracts in incentivizing asset managers. Brennan (1993) examines the CAPM implications of delegated management with both exogenous and optimal benchmarking. Admati and Pfleiderer (1997) analyzes benchmarking and manager incentives in a partial equilibrium framework in which managers have superior information to investors, while van Binsbergen et al. (2008) explores how benchmarking can overcome moral hazard issues that arise with decentralization. Cuoco and Kaniel (2011) study the implications for asset pricing when manager compensation is linked to a benchmark, and Li and Tiwari (2009) study nonlinear performance-based contracts in the presence of benchmarking. In our work, we derive the optimal benchmark jointly with the optimal affine contract and equilibrium prices, and study their empirical implications for intermediary holdings and asset returns.

2 A Model of Delegated Asset Management

In this section, we present a model of delegated asset management with flexible information acquisition in a multi-asset framework. We first introduce the asset environment, and then discuss the agency friction that fund managers face in portfolio allocation decisions. Finally, we define the asset market equilibrium.

2.1 The Environment

Asset Fundamentals There are three dates $t = \{0, 1, 2\}$. Suppose that there are N assets with risky payoffs f_i, $i \in \{1, 2, ..., N\}$, which realize at date 2 that satisfy the following decomposition:

$$f_i = \begin{cases} b_1\theta_1 \\ a_i\theta_i + b_i\theta_1, \ i \in \{2, ..., N\} \end{cases}$$

The common component θ_1 can be viewed as aggregate payoff risk, with b_i being the loading on this aggregate payoff risk of the asset, while the $a_i\theta_i, i \in \{2, ..., N\}$ are the asset-specific components of the risky asset payoffs. This payoff structure we employ is similar to that in Buffa et al. (2014) and Kacperczyk et al. (2016). For interpretation of θ_1 as aggregate payoff risk, we assume that $a_1 = 0, b_1 = 1$ and that the first asset is a composite asset of the remaining assets in the economy with a payoff that loads only on this aggregate payoff risk.[3] In addition to the N assets, there is a risk-free asset, which can be viewed as asset 0, in perfectly elastic supply with gross return $R^f > 1$. Asset i has price P_i at $t = 1$, and we stack the N prices into the $N \times 1$ vector $\mathbf{P}$. In what follows, bold symbols represent vectors.

[3] Kacperczyk et al. (2016) employ a similar assumption for the asset payoff structure. While not essential for our analysis, it helps with exposition by ensuring that the map from risk factors $\left\{\theta_1, \{\theta_i\}_{i\in\{2,...,N\}}\right\}$ to asset payoffs $\{f_i\}_{i\in\{1,...,N\}}$ is invertible.

For convenience, we define the vector $\Theta = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_N \end{bmatrix}'$ such that:

$$\mathbf{f} = F\Theta,$$

for the $N \times N$ matrix F, which is invertible since F is lower triangular provided that $b_i > 0$ $\forall i$. In our setting, aggregate risk arises through the correlation structure of asset payoffs, and is represented by the common fundamental θ_1.[4]

We assume that all agents in our model have a normal prior over Θ, and initially believe that $\Theta \sim \mathcal{N}\left(\bar{\Theta}, \tau_\theta^{-1} Id_N\right)$, where τ_θ is the common precision of the prior over the hidden factors driving asset payoffs. One can view the prior as reflecting all publicly available information about the asset payoffs, such as financial disclosures, earnings announcement, and macroeconomic news that agents have before contracting at date 0.

Agents There are two types of agents in the market: investors and managers. Both agents are risk averse with CARA preference over their consumption. Investors can invest directly in asset markets or delegate management of their portfolio to fund managers. At date 0 a fraction χ of investors delegate management of their portfolio to fund managers, and a fraction $1 - \chi$ manage their portfolio directly. The managers have skills, which means that they can exert unobservable effort to obtain private signals about asset fundamentals.[5] This is what we refer to as *delegated learning* channel. Each manager owns and operates one fund. The fraction χ is observable public information. Similar to van Binsbergen et al. (2008), we analyze the incentive contract between investors and managers by modeling the one layer delegation problem, i.e., investors directly offer compensation contracts to fund managers.[6]

[4]This is in contrast to Kacperczyk et al. (2016), where aggregate risk takes the form of the asset fundamental with a higher supply variance. Our derivations will, in fact, be valid more generally for any arbitrary invertible matrix F.

[5]Brown and Davies (2016) also studied the moral hazard in the active asset management industry in a partial equilibrium framework. They assume the effort exerted by managers are directly linked to returns, while we focus on the incentives to acquire costly information.

[6]As long as fund managers cannot commit to their future actions, it will not matter whether the investors or the managers offer the contract.

Our approach is different from García and Vanden (2009) and Gárleanu and Pedersen (2015), who investigate the fee setting by asset management companies. Given the relatively stable mutual fund fee structure and that the initial size of funds is fixed by assumption, we focus on studying the direct incentive provision from the compensation contract to managers. We discuss the problem faced by each of these agents in turn.

2.2 Fund Managers

Fund managers face a portfolio choice problem at date 1. Given their information and initial AUM W_0, fund managers choose a portfolio allocation strategy ω_1^S at date 1 across the N assets *after* observing market prices $\mathbf{P}$, so that the final AUM W_2^S is given by:

$$W_2^S = R^f W_0 + \omega_1^{S\prime} \left(\mathbf{f} - R^f \mathbf{P} \right).$$

In addition to a portfolio choice problem, fund managers also face an information acquisition choice. We assume managers must exert costly effort at date 0 to acquire information about asset payoffs at date 1.

While asset prices are publicly observable, managers acquire a vector of noisy private signals $\mathbf{s}_j$ about θ_1 and the asset-specific component of asset payoffs θ_i, $i \in \{2, ..., N\}$. They are able to exert effort $e = \mathbf{e}'\mathbf{1}_{N\times 1} \geq 0$, with $\mathbf{e} \geq \mathbf{0}$ element-by-element, to reduce the variance of these signals $\Sigma(\mathbf{e})$. Although investors are matched with fund managers, the level of effort that fund managers exert is not observable.

Fund manager j receives a vector of noisy signals $\mathbf{s}_j$ about Θ given the effort level $\mathbf{e}_j$:

$$\mathbf{s}_j = \Theta + \Sigma_j (\mathbf{e}_j)^{1/2} \varepsilon_j,$$

where $\varepsilon_j \sim N(\mathbf{0}_{N\times 1}, Id_N)$ is independent across j and satisfies the Strong LLN $\int_{-\infty}^{\infty} \varepsilon_j d\Phi(\varepsilon_j) = \mathbf{0}_{N\times 1}$ for $\Phi(\cdot)$, the CDF of the standard normal distribution. Following Kacperczyk et al. (2014), we assume that $\Sigma_j(\mathbf{e}_j)$ is a diagonal matrix with entry $K_{ii}^{-1}(e_{ij})$ that satisfies a monotonicity condition.[7] We assume that $\Sigma_j(\mathbf{e}_j)$ is diagonal so that there is a direct link between the effort manager j exerts to learn about the i^{th} component of Θ, $\mathbf{e}_{ij}$, and the precision of the signal manager j receives about that component, s_{ij}.[8] The monotonicity condition we impose ensures that a higher level of effort (weakly) implies the manager receives more informative signals. To ensure prices are always informative, we regulate $\Sigma(\mathbf{e}_j)$ by assuming that $\sup_i \Sigma(\mathbf{0}_{N\times 1}) \leq M^{-1} < \infty$.[9] In what follows, we choose the parameterization $K_{ii}(e_{ij}) = M + e_{ij}$. One can view this observation of private information by a fund manager as their security selection or "stock picking ability".

Fund managers have CARA preferences over their compensation from investors C_0^S and the monetary cost of exerting effort of information acquisition:

$$u\left(C_0^S; \omega_1^S, \mathbf{e}\right) = -\exp\left(H(\mathbf{e}) - \gamma_M C_0^S\right),$$

where γ_M is the coefficient of absolute risk aversion and $H(\cdot)$ is the dollar cost for effort $\mathbf{e}$, an increasing and (strictly) convex function in each of its arguments, such that $\partial_i H(\mathbf{e}) > 0$ and $\partial_{ii} H(\mathbf{e}) \geq 0$, and $H(\mathbf{0}_{N\times 1}) = 0$ as a normalization. We specialize $H(\mathbf{e})$ to the case that $H(\mathbf{e}) = \frac{1}{2} h(\mathbf{e}'\mathbf{1}_{N\times 1})$, where $h'(\cdot) > 0$, $h''(\cdot) \geq 0$, and $h(0) = 0$. This functional form induces substitutability in manager learning decisions, and therefore a tradeoff to learning too much about one source of asset-specific risk. Since fund manager effort is unobservable, a fund manager must find it optimal to choose the effort level recommended by the investor, which gives rise to the incentive compatibility (IC) constraint:

$$\mathbf{e} \in \operatorname{argsup}_{\mathbf{e}\in\mathbb{R}_+^N} E\left[\sup_{\omega\in\mathbb{R}^N} E\left[u\left(C_0^S; \omega_1^S, \mathbf{e}'\right) \mid \mathcal{F}_j\right]\right] \quad (IC), \tag{1}$$

where $\mathcal{F}_j$ is the fund manager's information set, and the optimization implies a natural tim-

[7] The monotonicity condition we require is that : $\Sigma_j(\mathbf{e}_j'') - \Sigma_j(\mathbf{e}_j')$ is positive-semi definite (PSD) whenever $\mathbf{e}_j' \geq \mathbf{e}_j''$.

[8] Our results are robust to the more general specification of $\Sigma_j(\mathbf{e}_j)$.

[9] Our results will be valid in the limit that $M \searrow 0$.

ing to their decisions. The fund manager first determines the effort to exert based on the compensation contract C_0^S with investors at date 0. At date 1, the fund manager observes prices and private signals, and makes portfolio allocation choice. The fund manager's information set is then the sigma algebra generated from observing the vector of prices $\mathbf{P}$ and its private signals $\mathbf{s}_j$, $\mathcal{F}_j = \sigma\left(\mathbf{P}, \mathbf{s}_j\left(\mathbf{e}_j\right)\right)$.

2.3 Delegating Investors

Investors have CARA preferences over the final AUM at date 2, W_2^S. They choose a compensation contract at date 0, C_0^S, for a manager to maximize their utility subject to incurring the cost of incentivizing the manager:

$$U\left(W_2^S, C_0^S\right) = -\exp\left(-\gamma\left(W_2^S - C_0^S\right)\right),$$

where $\gamma > 0$ is their coefficient of absolute risk aversion. Since investors only have access to public information, they have what we refer to as the common knowledge or public information set at $t = 1$, $\mathcal{F}^c$, which is the sigma algebra generated by observing prices $\mathcal{F}^c = \sigma\left(\mathbf{P}\right)$.

The investors solve the optimization problem when investing with managers:

$$V_0^S = \sup_{C_0^S} E^{\mathbf{e}\left(C_0^S\right)}\left[U\left(W_2^S, C_0^S\right)\right], \tag{2}$$

subject to the IR and IC constraints, where $E^{\mathbf{e}(\cdot)}\left[\cdot\right]$ is understood as the expectation under the probability distribution induced by the recommended effort level $\mathbf{e}\left(C_0^S\right)$. Consequently, V_0^S is the value of investing with a fund manager.

Similar to Kapur and Timmermann (2005), Buffa et al. (2014) and Sotes-Paladino and Zapatero (2017), we restrict our attention to the space of affine contracts between investors and fund managers.[10] A key reason is to advance our understanding of how the incentives

[10] In practice, investors pay fees to advisory firms who then compensate the managers through the incentive

faced by mutual fund managers extend to an equilibrium setting. Previous partial equilibrium studies have found negative results on affine incentive contracts for fund managers. Stoughton (1993) and Admati and Pfleiderer (1997) both show that affine contracts provide no incentives for fund manager effort, while regulations restrict the form of compensation contracts to only be symmetric around benchmark returns. SEC regulation restricts the form of compensation contracts for fund managers to fulcrum fees, which are symmetric around the returns of the fund's benchmark.[11] We analyze the optimal contract in the general linear setup and show that affine contracts can provide managerial incentives when asset prices that contain private information feed back into the compensation contracts.[12] In addition, since we are solving for noisy rational expectations within the linear paradigm of Grossman and Stiglitz (1980) and Hellwig (1980), such a restriction may be seen as a natural extension of the focus on linear equilibria.

2.4 Direct Investors

Direct investors allocate their capital W_0 directly in financial markets and have CARA preferences over the final AUM at date 2, W_2^D. They choose a portfolio ω_1^D at date 1 for their fund after observing the market prices $\mathbf{P}$, so that the final AUM W_2^D is given by:

$$W_2^D = R^f W_0 + \omega_1^{D\prime} \left(\mathbf{f} - R^f \mathbf{P} \right).$$

contracts. Since mutual fund fees are relatively stable over time (Pástor and Stambaugh, 2012), we focus on the manager incentive problem directly.

[11]The 1970 SEC amendment to the Investment Company Act of 1940 requires that performance-based contracts should not contain the "bonus" performance-based fee and should only be symmetric around the benchmark returns.

[12]Starks (1987) shows that linear contracts will lead to optimal portfolio risk exposure by managers, but an under-provision of effort compared to the first-best. As such, the contracts we characterize may potentially be suboptimal in incentivizing managers to acquire information. As Starks (1987) emphasizes, however, asymmetric contracts embedded with bonus incentives lead to an even lower level of effort than in the symmetric case, as well as suboptimal risk exposure.

Direct investors make investment decisions to maximize their:

$$V_0^D = \sup_{w_1^D} E[U\left(W_2^D\right)] = \sup_{w_1^D} E[-\exp\left(-\gamma\left(W_2^D\right)\right)],$$

where $\gamma > 0$ is their coefficient of absolute risk aversion. In the Internet Appendix, we provide a micro-foundation for direct investing in which investors delegate their portfolio choice decision to unskilled fund managers who are incapable of acquiring private information.

2.5 Free Entry and Asset Markets Clearing

Free Entry We assume that investors can freely choose to invest with a fund manager or to invest directly. Since there is a fixed fraction of funds available to investors, in equilibrium they must be indifferent between these two options. This implies that the indirect utility to investing with a fund manager V_0^S or to investing directly V_0^D must be equal, or

$$V_0^S = V_0^D.$$

This free-entry assumption is similar to that in Berk and Green (2004), where the "net fees" of funds with managers versus direct investing offers similar returns, while "gross of fees" reflects manager skill. Furthermore, Berk and van Binsbergen (2015) find that active managers capture the surplus in the advisory relationship, and this will show up as a fixed fee in our affine contract. Given that the investment decisions of fund managers will be independent of initial wealth in this CARA-normal setting, we are abstracting from the decreasing returns to scale at the fund level that are observed empirically, the consequences of which are explored, for instance, in Berk and Green (2004) and Pástor and Stambaugh (2012). Since investors in our model are indifferent to with whom they invest, the market for intermediation between investors and managers then trivially clears.

Market Clearing and Equilibrium Let $\omega_1^S(i)$ be the portfolio allocation of the fund manager $i \in [0,1]$, and similarly with $\omega_1^D(i)$ for direct investor i. Given that direct investors are atomistic, they will all follow the same portfolio strategy, $\omega_1^D(i) = \omega_1^D$. We assume the supply of the asset is given by the vector $\mathbf{x}$ for the N assets. Since there are a fraction χ of investors delegating their investment decisions to the fund managers, and a fraction $1-\chi$ of direct investors, market-clearing requires that:

$$\chi \int_0^1 \omega_1^S(i)\, di + (1-\chi)\, \omega_1^D = \mathbf{x}. \tag{3}$$

As is common in the literature, we assume that asset supply $\mathbf{x}$ is noisy to prevent beliefs from being degenerate.[13] We assume that, from the perspective of all agents, $\mathbf{x} \sim \mathcal{N}\left(\bar{\mathbf{x}}, \tau_x^{-1} Id_N\right)$ has a multivariate normal distribution, and $\bar{\mathbf{x}} > \mathbf{0}$ (element-by-element). Since all fund managers are atomistic, they take prices as given and each has negligible impact on the price formation process.

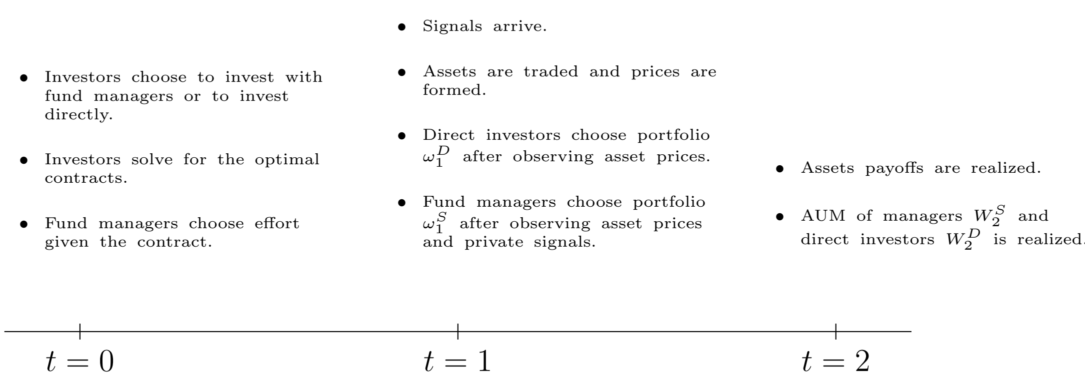


Figure 1: Timeline

Figure 1 illustrates the time line. We solve for a perfect Bayesian noisy rational expectations equilibrium defined as follows:

[13]In the absence of supply noise, beliefs between the common component of payoffs θ_1 and the asset-specific component $\theta_i, i \in \{2, 3, ..., N\}$ would have to be perfectly negatively correlated once prices $\mathbf{P}(\Theta)$ are observed, as a result of Bayesian updating. Since there are N hidden states Θ and only N assets, the vector price function $\mathbf{P}(\Theta)$ is a rank-deficient map from $\mathbb{R}^N$ to $\mathbb{R}^N$.

A perfect Bayesian noisy rational expectations equilibrium in this economy is a list of policy functions $\mathbf{e}\left(C_0^S\right)$, $\omega_1^S\left(\mathbf{s}_j, \mathbf{P}\right)$, and $\omega_1^D\left(\mathbf{P}\right)$, compensation contract C_0^S for fund managers, and prices $\mathbf{P}$ such that:

- Direct Investor Optimization: Given prices $\mathbf{P}$, and information set $\mathcal{F}^c$, $\omega_1^D\left(\mathbf{P}\right)$ satisfies each direct investor's IR constraint, and delivers expected utility V_0^D.

- Delegating Investor Optimization: Contract C_0^S solve the investor's optimization problem (2) and delivers expected utility V_0^S.

- Fund Manager Optimization: Given contract C_0^S, prices $\mathbf{P}$, and information set $\mathcal{F}_j$, $\mathbf{e}\left(C_0^S\right)$, and $\omega_1^S\left(\mathbf{s}_j, \mathbf{P}\right)$ solve each fund manager's IC constraint.

- Market Clearing: The asset markets clear through equation (3).

- Consistency: Investors (direct and delegating) form their expectations about Θ based on their information set $\mathcal{F}^c$, while fund managers form their expectations based on their information set $\mathcal{F}_j$, according to Bayes' rule.

- Sequential Rationality: For each realization of prices $\mathbf{P}$ and private signals $\mathbf{s}_j$, fund managers find it optimal at date 1 to follow investment policy $\omega_1^S\left(\mathbf{s}_j, \mathbf{P}\right)$.

3 The Equilibrium

We search for a symmetric linear equilibrium in which we conjecture that asset prices $\mathbf{P}\left(\Theta, \mathbf{x}\right)$ take the linear form:

$$\mathbf{P}\left(\Theta, \mathbf{x}\right) = \Pi_0 + \Pi_\theta \Theta + \Pi_x \mathbf{x}, \tag{4}$$

where $Rank\left(\Pi_\theta\right),\ Rank\left(\Pi_x\right) = N$. As discussed above, we also focus on linear contracts.

We first derive the conditional beliefs of investors and fund managers. We then derive the optimal investment policy for direct investors and then for fund managers, who face both effort and portfolio choice decisions that must be incentive compatible. Imposing market clearing allows us to solve for equilibrium asset prices. Finally, we solve for the optimal contracts offered by delegating investors to fund managers.

3.1 Learning

We begin by deriving the learning process for direct investors. Since direct investors have a normal prior, after observing the linear Gaussian signals $\mathbf{P}(\Theta)$, they update to a posterior for Θ that is also Gaussian $\Theta \mid \mathbf{P}(\Theta) \sim \mathcal{N}\left(\hat{\Theta}, \Omega\right)$ with conditional mean $\hat{\Theta}$ and conditional variance Ω, given by:

$$\hat{\Theta} = \Omega\tau_\theta\bar{\Theta} + \Omega\tau_x\Pi_\theta'\left(\Pi_x\Pi_x'\right)^{-1}\left(\mathbf{P} - \Pi_0 - \Pi_x\bar{\mathbf{x}}\right), \tag{5}$$

$$\Omega^{-1} = \tau_\theta Id_N + \tau_x\Pi_\theta'\left(\Pi_x\Pi_x'\right)^{-1}\Pi_\theta. \tag{6}$$

To get to the posterior of fund manager j, we recognize that we can first have the manager update his beliefs based on the publicly observed prices, and then treat these beliefs as an updated prior for when the manager then observes its vector of private signals $\mathbf{s}_j$. After observing the public signals $\mathbf{P}(\Theta)$, the new prior of fund manager j from above is $\Theta \mid \mathbf{P}(\Theta) \sim \mathcal{N}\left(\hat{\Theta}, \Omega\right)$, with $\hat{\Theta}$ and Ω given by equations (5) and (6), respectively.

After observing its vector of private signals, the posterior of fund manager j is also Gaussian $\Theta \mid \{\mathbf{P}(\Theta), \mathbf{s}_j\} \sim \mathcal{N}\left(\hat{\Theta}(j), \Omega(j)\right)$ with conditional mean $\hat{\Theta}(j)$ and the conditional variance $\Omega(j)$ summarized by the following two expressions:

$$\hat{\Theta}(j) = \Omega(j)\Omega^{-1}\hat{\Theta} + \Omega(j)\Sigma_j(\mathbf{e}_j)^{-1}\mathbf{s}_j, \tag{7}$$

$$\Omega(j)^{-1} = \Omega^{-1} + \Sigma_j(\mathbf{e}_j)^{-1}. \tag{8}$$

This completes our characterization of learning by direct investors and fund managers. Having solved for the conditional beliefs of all agents, we next analyze the optimal portfolio investment and effort policies of direct investors and fund managers.

3.2 Optimal Policies of Direct Investors

We begin our analysis of optimal policies with direct investors. Given investors have CARA preferences and payoffs are normally distributed, it follows that we can express the investor's optimization problem as:

$$V_0^D = \sup_{\omega_{\mathbf{1}}{}^D} R^f W_0 + \omega_{\mathbf{1}}{}^{D\prime} \left(F\hat{\Theta} - R^f \mathbf{P} \right) - \frac{\gamma}{2} \omega_{\mathbf{1}}{}^{D\prime} F \Omega F' \omega_{\mathbf{1}}{}^D,$$

given the properties of log-normal distributions and the monotonicity of the utility function in wealth. This optimization has the following straightforward interior solution:

$$\omega_{\mathbf{1}}{}^D = \frac{1}{\gamma} \left(F \Omega F' \right)^{-1} \left(F\hat{\Theta} - R^f \mathbf{P} \right), \tag{9}$$

which is consistent with mean-variance preferences in this normal setting. The superscript D indicates that this is the optimal investment portfolio for direct investors given information set $\mathcal{F}^c$.

We can also calculate the expected utility of direct investors, V_0^D. By the law of iterated expectations, first conditioning on $\mathcal{F}^c$, the expected utility to direct investors, V_0^D, is then:

$$V_0^D = -E \left[\exp \left(-\gamma \left(W_2^D \right) \right) \right] = -E \left[\exp \left(-\gamma R^f W_0 - \frac{1}{2} \mathbf{Z}' \Omega^{-1} \mathbf{Z} \right) \right],$$

where $\mathbf{Z} = \hat{\Theta} - R^f F^{-1} \mathbf{P}$ is the ex ante excess return to asset fundamentals, and $\mathbf{Z} \sim \mathcal{N} \left(\mu, \Omega_Z \right)$.[14]

3.3 Optimal Policies of Fund Managers

Fund managers must be incentivized since they add value to the investor's portfolio through their hidden, costly acquisition of private information. As such, they can no longer be perfectly monitored since they are free to choose $\mathcal{F}_j-$measurable portfolio strategies, $\omega_1^S =$

[14] See Appendix A.2 for the detailed proof.

$\omega_1^S(\mathbf{P}, \mathbf{s}_j)$, and $\mathcal{F}^c \subseteq \mathcal{F}_j$. Consequently, it is not generically possible for the investor to invert the private signals $\mathbf{s}_j$ the manager received from the realized portfolio excess payoff $W_2^S - R^f W_0$ to ensure that the manager followed the investor's recommendation contingent on observing signals $\mathbf{s}_j$. What is worse is that, even if the investor could observe $\mathbf{s}_j$ directly, the investor could not ex post verify that the fund manager exerted the recommended effort level $\mathbf{e}$ to obtain the desired precision of the signals.[15]

These considerations motivate us to consider compensation schedules C_0^S that are contingent on outcomes observable to investors at date 2 and, as such, we consider contracts that condition on the realized portfolio return per share of the fund $W_2^S - R^f W_0$ and the realized excess payoffs of the risky assets $\mathbf{f} - R^f\mathbf{P}$, $C_0^S = C_0^S\left(W_2^S - R^f W_0, \mathbf{f} - R^f\mathbf{P}\right)$.[16] We focus on linear contracts and conjecture the optimal contract C_0^S is in the form of

$$C_0^S = \rho_0 + \rho_S\left(W_2^S - R^f W_0\right) + \rho_R'\left(\mathbf{f} - R^f\mathbf{P}\right). \tag{10}$$

Conditioning compensation on the realized excess payoff of the portfolio potentially helps to align the incentives of the fund manager and investor by giving the manager an equity stake in the portfolio. This feature is similar to the fixed fraction of assets under management fee that mutual funds charge in practice, consistent with the recent finding by Ibert et al. (2017) using a unique data set of compensation on Swedish mutual fund managers. In addition, allowing the compensation schedule to vary with observed excess payoffs $\mathbf{f} - R^f\mathbf{P}$ can also improve incentives by providing flexibility for the contract to take into account realized market conditions through $\mathbf{f} - R^f\mathbf{P}$.

Since their effort and portfolio choice are unobservable, fund managers choose incentive

[15] In part, the assumption that the variance of private signals is regulated from above in the sup norm by $\frac{1}{M}$ would ensure that there are limits to monitoring low levels of effort by observing very extreme realizations of private signals.

[16] We also considered a version in which the compensation contract conditions on the realized return of direct investors W_2^D. Since W_2^D is based on public information, and is exogenous to the choices of any fund manager, it has no substantive impact on their information acquisition decisions. It does, however, affect the hedging incentives in their portfolio choice. In addition, tournament incentives are not prevalent in the asset manament industry. See Kapur and Timmermann (2005) for this type of incentive contract.

compatible portfolios that solve the inner optimization program (1). Conditional on this portfolio choice, which has both a mean-variance component and a hedge against the excess payoff portion of their contract, they choose their optimal effort to minimize the conditional variance of their excess payoff. This is summarized in Proposition 1.

Proposition 1 *The optimal portfolio of a fund manager* ω^S *is given by:*

$$\omega_1^S(j) = \frac{1}{\gamma_M \rho_S}\left(F\Omega(j)F'\right)^{-1}\left(F\hat{\Theta}(j) - R^f\mathbf{P}\right) - \frac{1}{\rho_S}\rho_R,$$

and the optimal level of effort $\mathbf{e}$ *satisfies:*

$$Diag\left[\left(\Omega^{-1} + M \cdot Id_N + diag(\mathbf{e})\right)^{-1}\right] \leq h'\left(\mathbf{e}'\mathbf{1}_{N\times 1}\right)\mathbf{1}_{N\times 1}, \tag{11}$$

where Diag is the diagonal operator. If F *is diagonal, then this condition further reduces to*

$$\frac{1}{\Omega_{ii}^{-1} + M + \mathbf{e}_i} \leq h'\left(\mathbf{e}'\mathbf{1}_{N\times 1}\right) \ \forall \ i \in \{1, ..., N\}. \tag{12}$$

From Proposition 1, the linear contract induces the fund manager to take the optimal mean-variance portfolio given its beliefs, with effective risk aversion $\gamma_M \rho_S$, corrected by a hedging position $-\frac{1}{\rho_S}\rho_R$ that takes into account that the manager is exposed to payoff risks $\mathbf{f} - R^f\mathbf{P}$ independent of the return on the portfolio he manages. The optimal level of effort $\mathbf{e}$ from equation (11) is determined only by the second moments of the conditional excess payoff $F\hat{\Theta}(j) - R^f\mathbf{P}$, and seeks to minimize $\Omega(j)$, since $\Omega^{-1} + \Sigma_j(\mathbf{e}_j)^{-1} = \Omega(j)^{-1}$ is the expression within the trace operator in the condition for optimality.

The correlation structure of asset payoffs F induces substitutability in learning across asset fundamentals Θ for fund managers, in addition to the ex post correlation in beliefs captured in Ω. Fund managers choose their effort recognizing that learning about asset-specific fundamental $\theta_i, i \in \{2, 3, ..., N\}$ also reveals information about the aggregate fundamental θ_1 through prices, which further reveals information about the other asset-specific fundamentals θ_j for $j \neq i$. In the special case that F is diagonal, the FOC for the optimal effort from Proposition 1 reduces to equation (12). As one can see from equation (12), the benefit to the fund manager for increasing effort becomes separable across assets $\frac{1}{\Omega_{ii}^{-1}+M+\mathbf{e}_i}$. With (weakly) convex costs to exerting effort, it then makes sense for the fund manager to allocate all its attention to the asset that reduces the conditional variance of its excess payoff the most. Consequently, one would expect corner solutions to the fund manager's optimal effort problem when F is diagonal, and for the manager to allocate its attention to the fundamentals for which he is able to equate its marginal benefit of learning with the marginal cost.

Having characterized the optimal policies of direct investors and fund managers, we solve for equilibrium asset prices by imposing market clearing. Appendix A.1 contains the solution of equilibrium asset prices.

We can then examine how different components of the linear contract ρ_0, ρ_S, and ρ_R impact the information acquisition choice of fund managers. Substituting equation of equilibrium asset prices (A1) into equation (11) from Proposition 1, we can find the equilibrium level of effort exerted by fund managers in a symmetric equilibrium:

$$Diag\left[\left(\begin{array}{c}(\tau_\theta+M)\cdot Id_N+\tau_x\left(\frac{\chi}{\gamma_M\rho_S}\right)^2(M\cdot Id_N+diag\left(\mathbf{e}\right))\cdot\\ \left(F'F\right)^{-1}(M\cdot Id_N+diag\left(\mathbf{e}\right))+diag\left(\mathbf{e}\right)\end{array}\right)^{-1}\right]\leq\ h'\left(\mathbf{e}'\mathbf{1}_{N\times1}\right). \tag{13}$$

Importantly, it is the investor's choice of the sensitivity of the fund manager's compensation to the fund's return ρ_S that determines how the contract impacts the managerial incentives to acquire private signals, along with the manager's risk aversion γ_M and parameters that characterize the conditional uncertainty of asset payoffs given prices. With this condition characterizing the optimal effort of the fund manager in equilibrium, we can perform several comparative statics with equation (13) to understand how optimal effort changes with different features of the economic environment, taking into account that changes in effort change the informational content of prices. These comparative statics are summarized in Proposition 2.

Proposition 2 *The optimal choice of fund manager effort* $\mathbf{e}$, *in equilibrium, is increasing (element-by-element) in the coefficient of manager risk aversion,* γ_M*, and the sensitivity of manager compensation to its realized portfolio return,* ρ_S*. It is decreasing in the precision of the prior on* Θ, τ_θ*, the precision of the prior on the liquidity trading* $\mathbf{x}, \tau_x$*, and the fraction of fund managers,* χ*.*

From Proposition 2, in equilibrium, the sensitivity of the manager's compensation, ρ_S, increases the effort that the fund manager exerts to learn about the payoffs of risky assets. Intuitively, the more the manager's compensation depends on the fund's excess payoff, the more incentive the manager has to acquire information to improve the fund's performance. Similarly, the more effectively risk-averse the manager (higher γ_M), the less aggressively it trades on its private information and the less informative asset prices are, in equilibrium. This increase the benefit of learning, and causes the manager to exert more effort to reduce the uncertainty of the fund's final AUM. In addition, as one would expect, the more uncertain the economic environment (lower $\tau_\theta,\ \tau_x$, and χ), the more beneficial for the manager to exert effort to achieve the performance objectives of the fund.

3.4 The Optimal Affine Contract

We now focus on the optimal linear contract C_0^S that investors offer to the fund managers. Having solved for the determinants of optimal fund manager effort, in equilibrium, we provide a characterization of the optimal linear contract, which is summarized in Proposition 3.

Proposition 3 *The optimal affine contract for a fund manager is a* $N + 2 \times 1$ *vector* (ρ_0, ρ_S, ρ_R) *that sets*

$$\rho_R = -\left(\rho_S + \frac{\gamma}{\gamma_M}(1-\rho_S)\right)\omega^0,$$

where

$$\omega^0 = \frac{1}{\gamma}F'^{-1}Var\left(\Theta - R^f F^{-1}\mathbf{P}\right)^{-1} E\left[\Theta - R^f F^{-1}\mathbf{P}\right] = \frac{1}{\gamma}F'^{-1}\left(\Omega_Z + \Omega\right)^{-1}\mu,$$

is the ex ante mean-variance efficient portfolio, and

$$\rho_0 = \frac{1}{\gamma}\log\frac{V_0^D}{v^S},$$

where v^S is given in the Appendix. Furthermore, the optimal sensitivity on the realized excess payoff of the fund manager's fund ρ_S satisfies the FONC (A5) given in the Appendix.[17]

To help explore the implications of Proposition 3, we rewrite the optimal linear contract for a fund manager as:

$$\begin{aligned} C_0^S &= \frac{1}{\gamma}\log\frac{V_0^D}{v^S} + \rho_S\omega_1^S(i)'\left(\mathbf{f} - R^f\mathbf{P}\right) - \left(\rho_S + \frac{\gamma}{\gamma_M}(1-\rho_S)\right)\omega^{0\prime}\left(\mathbf{f} - R^f\mathbf{P}\right) \\ &= \frac{1}{\gamma}\log\frac{V_0^D}{v^S} + \rho_S\left(\omega_1^S(i)' - \omega^{0\prime}\right)\left(\mathbf{f} - R^f\mathbf{P}\right) - \frac{\gamma}{\gamma_M}(1-\rho_S)\,\omega^{0\prime}\left(\mathbf{f} - R^f\mathbf{P}\right) \end{aligned}$$

The first piece of the contract is a constant fee that ensures that, net fees, investors are indifferent between investing with fund managers and direct investing. The second piece is the manager's compensation based on the fund's performance relative to the ex ante mean-variance portfolio ω^0. The third adjusts compensation by the performance of an index that tracks the ex ante mean-variance efficient portfolio investors would choose at the time that the contract is signed. Essentially, compensation beyond a fixed fee is offered for the value added by the manager over the investment strategy that investors could achieve through direct investment without acquiring any public or private information.

Notice that ω^0 plays the role of a passive benchmark for fund manager compensation, since it is a portfolio whose holdings are chosen based on only public information at $t = 0$.[18] As such, benchmarking is a feature of optimal contracting for delegated asset management with asymmetric information. Under certain conditions, this passive portfolio is also featured

[17] Substituting for Ω with equation (A1), $\mathbf{e}_j$ with equation (13), and ρ_R from Proposition 3 into the FONC (A5), we can then solve for the fixed point to find the equilibrium value of ρ_S.

[18] The ex ante mean-variance portfolio will also be the market portfolio if trading is allowed to occur at $t = 0$, since all investors and managers are initially identical. As such, one can view the benchmark as the market portfolio in a CAPM world.

as an optimal benchmark in Admati and Pfleiderer (1997).[19] The sensitivity of manager compensation to this benchmark, $-\frac{\gamma}{\gamma_M}(1-\rho_S)$, is intimately linked to the sensitivity of manager compensation to the fund's performance, ρ_S. The more risk-averse is the investor relative to the manager (higher $\frac{\gamma}{\gamma_M}$), the greater the magnitude of the sensitivity of the manager's compensation to the benchmark portfolio over $1-\rho_S$, since $\rho_S \in [0,1]$. This is similar to the optimal benchmark in van Binsbergen et al. (2008), which features a tilt that corrects for differences in risk attitudes between the fund manager and the delegating CIO, in addition to the minimum variance portfolio.

To further understand the impact of incentives on a fund manager's actions, we rewrite the optimal portfolio choice of the fund manager by substituting for ρ_R:

$$\omega_1^S(j) = \frac{1}{\gamma_M \rho_S}\left(F\Omega(j)F'\right)^{-1}\left(F\hat{\Theta}(j) - R^f\mathbf{P}\right) + \left(1 + \frac{\gamma}{\gamma_M}\left(\frac{1}{\rho_S} - 1\right)\right)\omega^0. \tag{14}$$

The portfolio of fund managers essentially have two components: a mean-variance efficient portfolio with respect to the manager's information set, and a long position in ω^0. Since fund manager's compensation is tied to the benchmark portfolio ω^0, they are effectively endowed with a negative exposure to the benchmark portfolio, and take a long position in ω^0 to hedge themselves. This benchmark-driven demand causes managers to over-invest in assets that are representative in their benchmark portfolio, and we refer this demand as *hedging* demand. This hedging channel is also a feature in Cuoco and Kaniel (2011), Basak and Pavlova (2013), and Buffa et al. (2014). In contrast to models in which benchmarking is assumed in the preferences of investors, such as in Basak and Pavlova (2013) and Duarte et al. (2015), in our model, the benchmark enters into security selection through the hedging demand of fund managers.

Fund managers here only care about benchmarking insofar as it affects their compensa-

[19] Admati and Pfleiderer (1997) identify the global minimum variance portfolio, tilted by the assets held by investors in separate accounts, as the optimal benchmark in a partial equilibrium setting. We derive benchmarking against the ex ante mean-variance efficient portfolio as a feature of the optimal affine compensation structure for fund managers with market-clearing. Since prices are determined by market-clearing, the benchmark portfolio is, itself, an equilibrium object that depends on the optimal contract.

tion, and this leads to the sterilization of the benchmark in the manager's optimal portfolio. In addition, the sensitivity of the contract to fund performance ρ_S is the transmission channel through which investors influence managerial effort to acquire information, rather than the benchmarking aspects of the compensation contract. Intuitively, if the fund manager deviates from the benchmark portfolio, it will choose the most profitable tilted portfolio based on its private information.

This completes our characterization of the perfect Bayesian noisy rational expectations equilibrium.

4 Model Implications

In this section, we discuss several empirical implications of our analysis. We begin by investigating the behavior of intermediaries. We then turn to the asset pricing implications of our framework, with an emphasis on predictions for the cross-section of asset returns.

4.1 Implications for Intermediaries

Since both the choice of benchmark portfolios and the skill of fund managers are endogenous and vary with respect to the fundamentals, it allows us to offer empirical predictions without conditioning on actual compensation contracts and observing managerial effort. By relating characteristics of the asset fundamentals to potentially observable fund outcomes such as their holdings and performance through the incentive contracts, our model also provides the theoretical link between the unobservable effort (skill) of the fund manager and the cross sections of fund behavior.

We consider a numerical example with two assets to illustrate our predictions. We choose as our baseline specification:

$$F = \begin{bmatrix} 1 & 0 \\ b & \sqrt{1-b^2} \end{bmatrix}, \ \bar{\Theta} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \bar{\mathbf{x}} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

In our discussion in this section, we refer to the asset whose payoff depends only on the aggregate fundamental θ_1 as Asset 1, and the asset that also has an asset-specific fundamental θ_2, with loading b on θ_1, as Asset 2. The F matrix is set to ensure that the comparative static of b is implemented keeping the level of uncertainty constant. Finally, we choose the effort function $h(\cdot)$ to be linear in effort $\mathbf{e}'\mathbf{1}_{2\times1}$, $h(\mathbf{e}'\mathbf{1}_{2\times1}) = \mathbf{e}'\mathbf{1}_{2\times1}$, so that the marginal cost of learning is constant. As a result, any substitutability in learning arises from the co-variance structure of asset prices. Although we consider a two-asset example for ease of exposition, we find that our results hold more generically.[20]

4.1.1 Optimal Effort and Portfolios

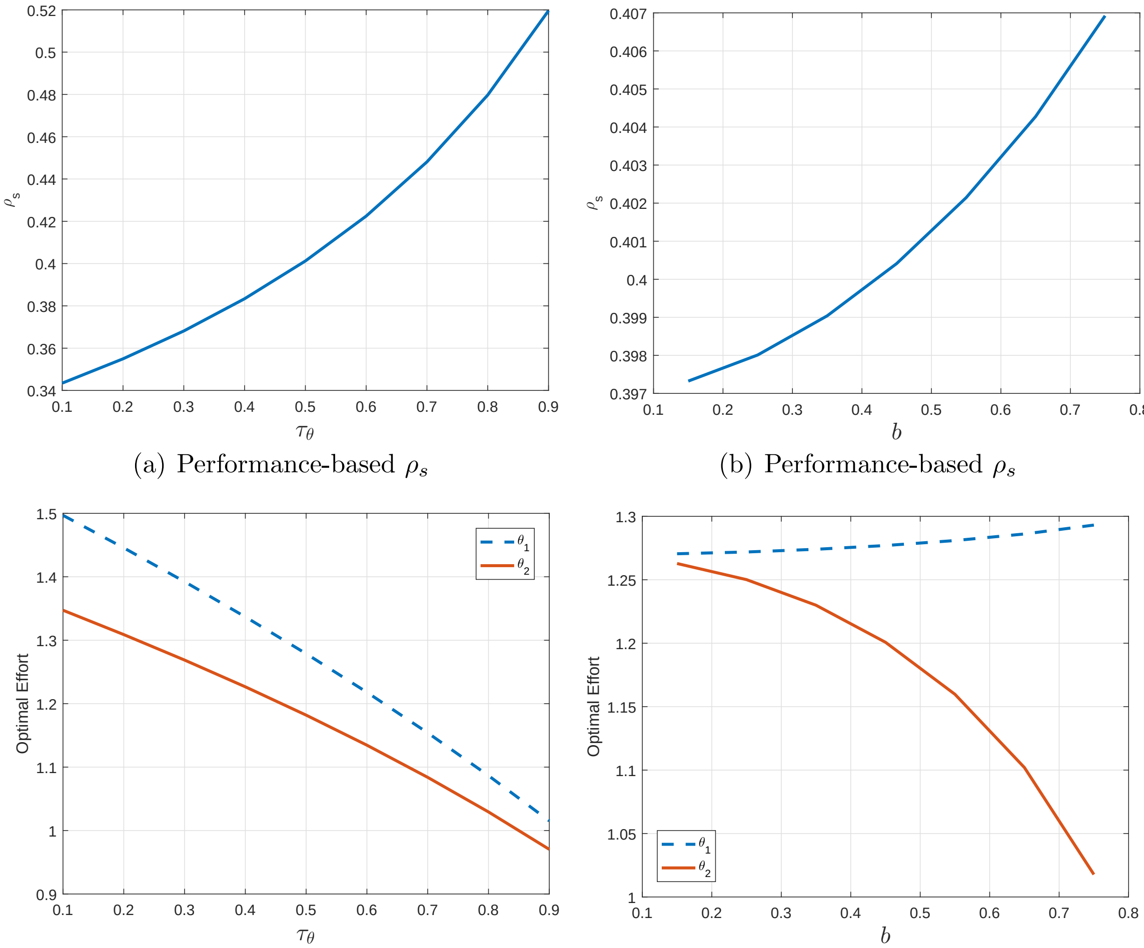
The affine incentive contract has two key features: performance-based incentive ρ_s and benchmarking. ρ_s determines how the contract impacts the manager's incentive to acquire private information. Panels (a) and (c) in Figure 2 show that as the ex ante uncertainty of asset payoffs τ_θ^{-1} declines, there are less benefits for the fund manager to acquire information. The optimal contract then puts more weight on the performance-based component ρ_s to provide incentives, and ρ_s increases with respect to τ_θ. Since we allow a general structure for asset payoffs, varying the correlation between asset payoffs also indicates a shift of incentives, as shown in Panels (b) and (d) of Figure 2. As b increases, the individual asset payoffs are more correlated with the aggregate component. Hence, the marginal benefit of learning about asset-specific information is low, while learning about the aggregate component has higher marginal benefit. Since information about aggregate component becomes more important as two assets are more correlated, the optimal contract provides more incentives to acquire

[20] We find qualitatively similar results for a 30-asset case available in Internet Appendix D.

private information by increasing ρ_s.

Figure 2: **Performance-based Compensation and Optimal Effort**

Parameters: $\tau_\theta = 0.5$, $\tau_x = 1$, $b = 0.5$, $\gamma_M = 2$, $M = 0$, $\chi = 0.3$, $\gamma = 1, W_0 = 1, R^f = 1.02$



(a) Performance-based ρ_s

(b) Performance-based ρ_s

(c) Optimal Effort $\mathbf{e}$

(d) Optimal Effort $\mathbf{e}$

Benchmarking rises endogenously in the optimal contract. To understand the active fund holdings through the hedging channel, we conduct a comparative statics analysis with respect to τ_θ and b. Figure 3 and Figure 4 show the holdings of both fund managers and direct investors, and the composition of the benchmark portfolio. As τ_θ increases, the payoffs of risky assets are less uncertain, and the representation of risky assets in the benchmark portfolio increases. This is because the benchmark is determined based on the risk-return trade off of the assets ex ante. Since fund managers' compensation is benchmarked to ω_0, they tend to hedge the benchmark risk by longing[21] the assets that are representative in the benchmark portfolio. This hedging behavior drives up the equilibrium prices of the assets that are representative in the benchmark. The change of correlation structure b impacts the benchmark portfolio as well. As two assets become more correlated, there is less diversification benefit in the ex ante mean-variance portfolio. Hence, as b increases in Figure 4 Panel (b), the holdings of both risky Assets 1 and 2 decline in the benchmark portfolio.

Hedging demand through general equilibrium effects is the key to understanding the implications of benchmarking on the active fund holdings. The expected excess payoffs to the fundamentals $E\left[\Theta - R^f F^{-1}\mathbf{P}\right]$ contain an additional component:

$$-\frac{1}{R^f}\chi\left(1+\frac{\gamma}{\gamma_M}\left(\frac{1}{\rho_S}-1\right)\right)\left(\left(\frac{1-\chi}{\gamma}+\frac{\chi}{\gamma_M\rho_S}\right)\Omega^{-1}+\frac{\chi}{\gamma_M\rho_S}\Sigma_j\left(\mathbf{e}_j\right)^{-1}\right)^{-1}F'\omega^0,$$

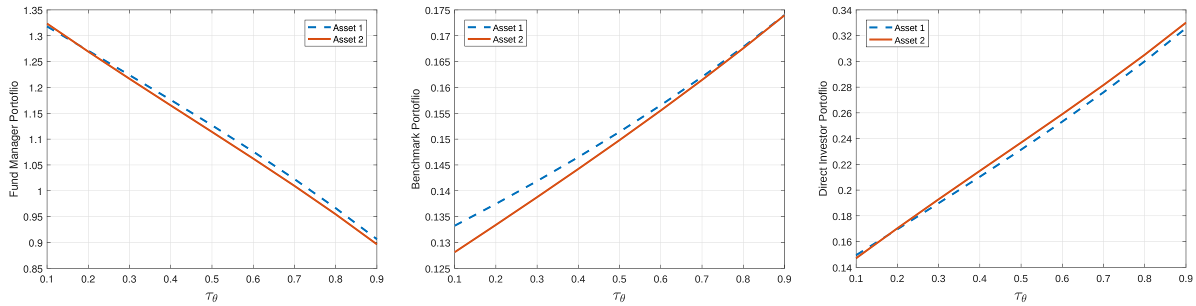
that reflects the risk of the fund manager's benchmark. If the manager pushes up prices because of hedging demand for assets that are more representative in the benchmark portfolio, this lowers the expected payoff. This effect helps fund managers, however, because they are evaluated relative to the performance of the benchmark, which now has a lower expected payoff. This is a general equilibrium effect through which the asset prices that are most inflated by benchmarking are those that are the largest constituents of the benchmark portfolio. As a consequence, fund managers both hedge themselves in their portfolios against the benchmark, and benefit from the benchmark's lower expected payoff from their aggregate hedging demand, as shown in Panels (a) and (c) in Figure 3.

One may wonder what impact benchmarking has on the investment performance of direct investors. Since fund managers all have a hedging demand for their exposure to the benchmark in their compensation, the liquidity providers for this demand are the direct investors. These direct investors are compensated by tilting their portfolios away from the benchmark, and toward assets that offer relatively higher excess payoffs. Consequently, direct investors benefit from benchmarking in fund manager compensation, since they earn risk premia for

[21] ρ_R is negative since the managers are compensated relative to the benchmark.

Figure 3: **Portfolios and Benchmark:** τ_θ

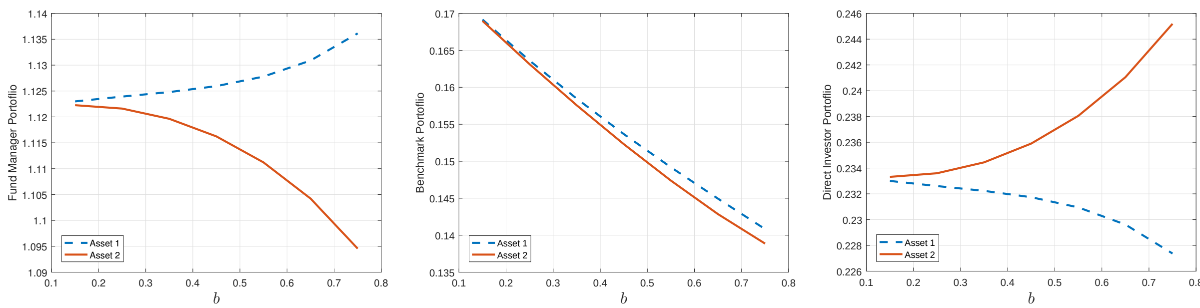
Parameters: $\tau_\theta = 0.5$, $\tau_x = 1$, $b = 0.5$, $\gamma_M = 2$, $M = 0$, $\chi = 0.3$, $\gamma = 1, W_0 = 1, R^f = 1.02$



(a) Fund Manager's Portfolio ω_S (b) Benchmark Portfolio ω^0 (c) Direct Investor's Portfolio ω_U

Figure 4: **Portfolios and Benchmark:** b

Parameters: $\tau_\theta = 0.5$, $\tau_x = 1$, $b = 0.5$, $\gamma_M = 2$, $M = 0$, $\chi = 0.3$, $\gamma = 1, W_0 = 1, R^f = 1.02$



(a) Fund Manager's Portfolio ω_S (b) Benchmark Portfolio ω^0 (c) Direct Investor's Portfolio ω_U

insuring fund managers against their benchmark exposure.

4.1.2 Performance Evaluation

We next show our central results of examining the link between our model-implied measure of skills and empirical statistics meant to capture asset manager's ability. In our framework, fund manager skill is the optimal effort level given the manager's incentive contract, and we define our model-implied measure as the decrease in uncertainty about asset payoffs $|\Omega| - |\Omega(j)|$.[22] Given the optimal decision, fund managers choose how much their portfolios deviate from the endogenous benchmark portfolio. To relate the effort exerted by the fund manager to the active share that is first introduced by Cremers and Petajisto (2009), we

[22]Our results are quantitatively similar if we use other model-implied measures of skill, such as $\mathbf{e_j}$ or $|\Omega - \Omega(j)|$.

define our active share as the deviation of a fund manager's portfolio holdings from the benchmark portfolio. The change in fundamental risk impacts the active share not just from shifting the learning incentives, but also from rebalancing the benchmark portfolio ex ante.

We derive an analogous expression for the average active share of a fund manager in our economy AS:

$$AS = \frac{1}{2} E\left[\mathbf{1}' \left|\omega_1^S(j) - \omega^0\right|\right],$$

where ω^0 is the benchmark for the fund manager.[23] Substituting for $\omega_1^S(j)$ with equation (14), ω^0 with Proposition 3, $\hat{\Theta}(j)$ with equation (7), and Π_θ and Π_x with equations (A2) and (A3), respectively, we can employ results for the expectation of a folded normal distribution to arrive at:

$$AS = \frac{1}{2\gamma_M \rho_S} \sum_{i=1}^{N} \left(\sqrt{\frac{2}{\pi}} \sigma_i e^{-\mu_i^2/2\sigma_i^2} + \mu_i \left(1 - 2\Phi\left(-\frac{\mu_i}{\sigma_i}\right)\right)\right),$$

where:

$$\begin{aligned}
\mu_i &= f_i' \left(\Omega^{-1} + \Sigma_j(\mathbf{e}_j)^{-1} + (1-\rho_S)(\Omega_Z + \Omega)^{-1}\right)\mu, \\
\sigma_i^2 &= f_i' \left(\Gamma_\theta \tau_\theta^{-1} \Gamma_\theta' + \Gamma_x \tau_x^{-1} \Gamma_x + \Sigma_j(\mathbf{e}_j)^{-1}\right) f_i,
\end{aligned}$$

where f_i is the i^{th} column of F^{-1}, and

$$\begin{aligned}
\Gamma_\theta &= \tau_x \left(\frac{\chi}{\gamma_M \rho_S}\right)^2 \Sigma_j(\mathbf{e}_j)^{-1} (F'F)^{-1} \Sigma_j(\mathbf{e}_j)^{-1} - R^f \left(\Omega^{-1} + \Sigma_j(\mathbf{e}_j)^{-1}\right) F^{-1}\Pi_\theta + \Sigma_j(\mathbf{e}_j)^{-1}, \\
\Gamma_x &= \left(\tau_x \Sigma_j(\mathbf{e}_j)^{-1} (F'F)^{-1} \Sigma_j(\mathbf{e}_j)^{-1} - R^f \left(\frac{\gamma_M \rho_S}{\chi}\right)^2 \left(\Omega^{-1} + \Sigma_j(\mathbf{e}_j)^{-1}\right) F^{-1}\Pi_\theta\right) \Sigma_j(\mathbf{e}_j) F'.
\end{aligned}$$

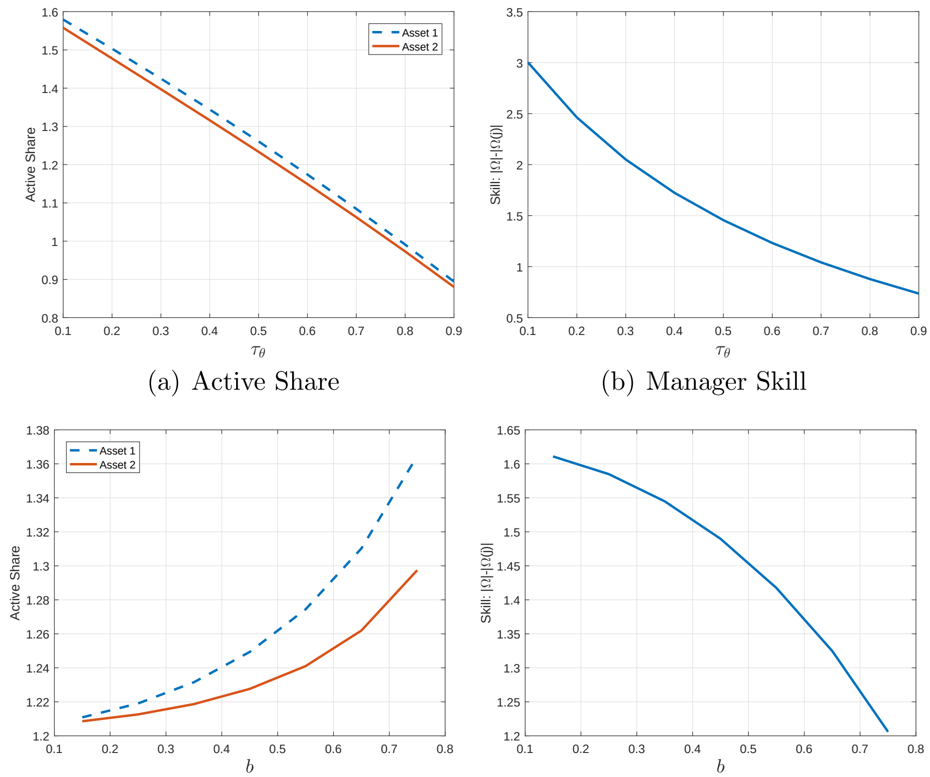
Figure 5 shows the comparative statics of the theoretical active share and the model-implied measure of skill $|\Omega| - |\Omega(j)|$ with respect to the change of τ_θ and b. As the uncertainty of asset

[23] One may notice that the definition of active share includes fund leverage. Since the benchmark can also take leveraged positions, this ensures an equitable comparison of portfolios when measuring manager activeness.

payoffs goes up (i.e., τ_θ decreases), the fund manager exerts more effort to learn and acquires more private information. Hence, both the benefit of learning and the measure of manager skills increases as τ_θ decreases, as shown in Figure 5 Panel (b). By acquiring additional

Figure 5: **Active Share and Fund Manager Skill: τ_θ & b**

Parameters: $\tau_\theta = 0.5$, $\tau_x = 1$, $b = 0.5$, $\gamma_M = 2$, $M = 0$, $\chi = 0.3$, $\gamma = 1, W_0 = 1, R^f = 1.02$



(a) Active Share (b) Manager Skill

(c) Active Share (d) Manager Skill

private information about asset fundamentals, the fund manager's portfolio further deviates from their benchmark portfolio, ω^0. Similarly, when asset fundamentals are less uncertain (higher τ_θ), the manager acquires less private information and takes a more passive position in financial markets.

We then obtain comparative statics for fund manager skill as the correlation b between the asset payoff increases (Figure 5 Panel (d)) through the incentive channel. The active shares of the two assets, however, go up. This occurs because the change in the correlation between two assets impacts the ex ante choice of the mean-variance portfolio (i.e., the benchmark portfolio). The benchmark portfolio reduces its weight in both Asset 1 and Asset 2 as there is less benefit from diversification when risky assets are more correlated. Although fund managers invest less in Asset 2, which is the asset with both aggregate and asset-specific components in its payoff, the decline in the weight of Asset 2 in the portfolio is not as much as the reduction in the benchmark portfolio.

Our prediction on the relation between a fund's active share and its benchmark also aligns with the critique of Frazzini et al. (2016). The active share measure can deviate from the underlying level of manager skill, since the incentives for the fund manager to learn are not strong enough to dominate the changes in the ex ante choices for benchmark portfolio. The incentives for the fund manager to acquire private information are shaped by the asset environment, so the effort exerted by the manager is correlated with its benchmark. While funds that invest in more volatile stocks are and appear more active, funds that invest in more correlated stocks may only appear more active because of their benchmark. Our prediction is also consistent with Jiang and Sun (2014), who studied dispersion in fund managers' beliefs about future stock returns based on their active holdings. The degree of information asymmetry is positively correlated with the dispersion of active mutual funds holdings under our delegated learning channel, since the incentives for fund managers to learn rises as the degree of uncertainty of asset payoffs increases, which is proxied by the idiosyncratic volatility of stock returns in Jiang and Sun (2014).

Our setting also allows us to explore another empirical measure for unobservable mutual fund actions, return gap (RG), employed in Kacperczyk et al. (2008). We rewrite the fund manager's portfolio as:

$$\omega_1^S(j) = \frac{\gamma}{\gamma_M \rho_S}\omega_1^D + \left(1 + \frac{\gamma}{\gamma_M}\left(\frac{1}{\rho_S} - 1\right)\right)\omega^0 + \frac{1}{\gamma_M \rho_S} F'^{-1}\Sigma_j \left(\mathbf{e}_j\right)^{-1}\left(\mathbf{s}_j - R^f F^{-1}\mathbf{P}\right).$$

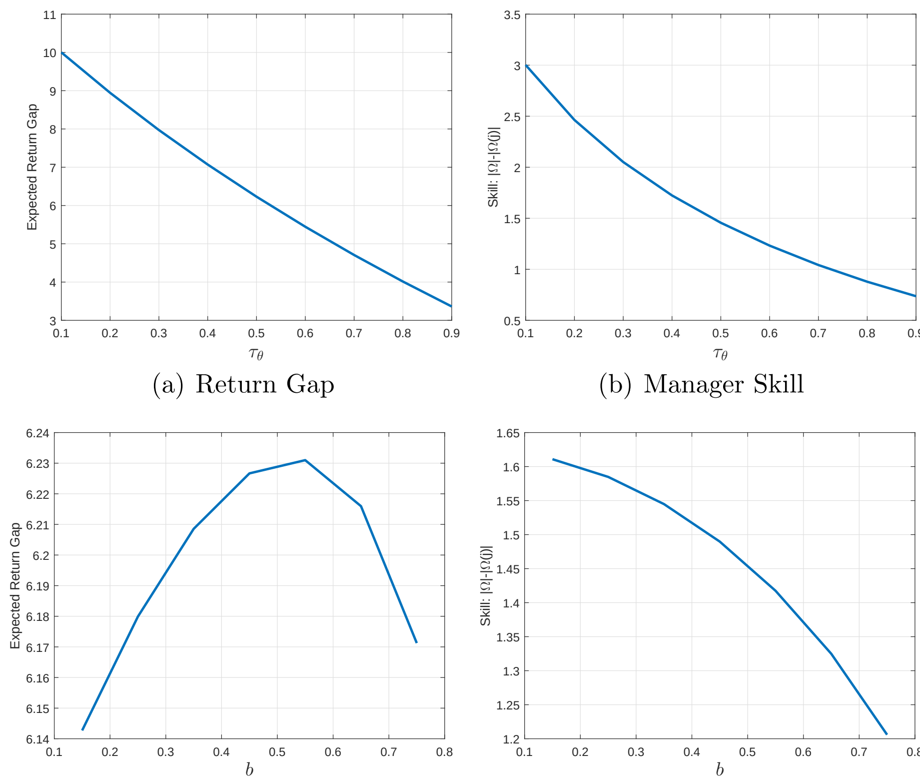
The first two elements reflect the position a fund manager without private information would take based on public information and the benchmark portfolio, while the last element captures the speculative bet the fund manager makes based on his informational advantage after observing its private signals. Consequently, we view the first two elements as the "holdings" portfolio that is publicly observable to investors, and measure the expected return gap between the gross return a fund manager garners and that of this "holdings" portfolio, RG. We can then construct the expected return gap:

$$E\left[RG\right] = \frac{1}{\gamma_M \rho_S} \mu' \Sigma_j \left(\mathbf{e}_j\right)^{-1} \mu + \frac{1}{\gamma_M \rho_S} Tr\left[\Sigma_j \left(\mathbf{e}_j\right)^{-1} \left(\Omega_Z + \Omega\right)\right].$$

The expected return gap is driven by fund managers trading more aggressively to collect the risk premia on assets since they face less risk because of their private information, and from the reduction in overall uncertainty they have when speculating.

Figure 6: **Return Gap and Fund Manager Skill: τ_θ & b**

Parameters: $\tau_\theta = 0.5$, $\tau_x = 1$, $b = 0.5$, $\gamma_M = 2$, $M = 0$, $\chi = 0.3$, $\gamma = 1, W_0 = 1, R^f = 1.02$



(a) Return Gap (b) Manager Skill

(c) Return Gap (d) Manager Skill

Figure 6 shows the comparative statics of the expected return gap and the model-implied measure of fund manager skill. The return gap is moving in the same direction as the skill measure of our model when the uncertainty level of the fundamental τ_θ^{-1} declines. The return gap, however, does not monotonically decrease in the correlation of asset fundamentals. As the correlation between asset fundamentals increases, there is less overall benefit to learning since the asset-specific fundamental is less relevant to returns, and the aggregate fundamental is more revealed by prices to both fund managers and direct investors. The expected return gap, in contrast, trades off two competing forces. On the one hand, there is increased risk in asset markets because a higher correlation among asset returns reduces the diversification benefit to holding both assets. Since they have access to private information, fund managers take larger exposures to the risky assets than direct investors. Hence, the risk premia earned by fund managers who bear more risk is higher through learning. On the other hand, the increased correlation also reveals more information to direct investors about the aggregate asset fundamental, reducing the information asymmetry between fund managers and direct investors. These two forces contribute to the humped-shaped return gap in Panel (c) in Figure 6.

We can also evaluate fund manager performance by computing expected excess returns in our setting, and compare them across the benchmark portfolio and both fund managers and direct investors. Given the benchmark portfolio ω^0 in Proposition 3, it is straightforward that a fund manager with portfolio that has an initial wealth W_0 who invests in the benchmark portfolio with final wealth W_2^0 will have an expected excess return:

$$E\left[W_2^0 - R^f W_0\right] = \frac{1}{\gamma}\mu' \left(\Omega_Z + \Omega\right)^{-1} \mu.$$

Making use of properties of chi-squared random variables, and that the trace operator is linear and satisfies $Tr\left[ABC\right] = Tr\left[BCA\right]$, we arrive at the expected excess return for direct investors:

$$E\left[W_2^D - R^f W_0\right] = \frac{1}{\gamma}\mu'\Omega^{-1}\mu + \frac{1}{\gamma}Tr\left[\Omega^{-1}\Omega_Z\right],$$

and similarly for a fund manager:

$$\begin{aligned} E\left[W_2^S - C_0^S - R^f W_0\right] &= E\left[W_2^0 - R^f W_0\right] + \frac{1}{\gamma_M}\left(\frac{1}{\rho_S} - 1\right)\mu'\left((\Omega_Z + \Omega)^{-1} + \Omega^{-1} + \Sigma_j\left(\mathbf{e}_j\right)^{-1}\right)\mu \\ &\quad + \frac{1}{\gamma_M}\left(\frac{1}{\rho_S} - 1\right) Tr\left[\left(\Omega^{-1} + \Sigma_j\left(\mathbf{e}_j\right)^{-1}\right)\Omega_Z + \Sigma_j\left(\mathbf{e}_j\right)^{-1}\Omega\right] - \rho_0. \end{aligned}$$

Kacperczyk et al. (2016) highlights that the additional return that a fund manager earns arises from their information acquisition decisions. As volatility falls in our setting, the expected excess return of both direct investors and the benchmark portfolio increases, reflecting the decreased uncertainty in investing and the more liquidity that their portfolios provide. In contrast, the expected excess return of fund managers falls as their superior information degrades. As one may expect, the first piece of fund returns is the benchmark portfolio's return.

4.1.3 Decline in Fund Flow and Performance in Active Management

Since 2006, over 90% of U.S. actively managed equity funds failed to beat their benchmark net of fees. During this time, they have also lost fund flows to passive strategies, both domestically and globally. A potential explanation for these phenomena is that there has been a downward trend in the level of skill among active asset managers. As can be seen in (Figure 5 (d)), our model predicts that a higher (pairwise) correlation between assets reduces the level of effort that fund managers exert to acquire private information. Cotter et al. (2016), among others, document a pronounced increase in the level of integration within and among assets classes, and across countries, since the 2008 financial crisis. Such an increase in asset correlations can, consequently, explain why fund managers have underperformed in recent years if their compensation contracts did not adjust to the new asset environment. Consistent with this view, much of the recent debate has been about reforming the incentive structure for the asset management industry.

4.2 Implications for Asset Pricing

Our equilibrium setup allows us to investigate predictions for asset returns. To facilitate our discussion, we first derive prices in the special case in which there are no fund managers, or $\chi = 0$. One can then show, in this "no information" setting, that prices $\mathbf{p}$ take the following linear form:

$$\mathbf{p} = \frac{1}{R^f} F\bar{\Theta} - \frac{1}{R^f}\gamma\tau_\theta^{-1} F F'\mathbf{x}.$$

Prices in this setting reflect the prior beliefs of investors about the payoff fundamentals $\bar{\Theta}$, and the net asset supply $\mathbf{x}$. The realized excess payoffs to the fundamentals Θ take the form:

$$\Theta - R^f F^{-1}\mathbf{p} = \Theta - \bar{\Theta} + \gamma\tau_\theta^{-1} F'\mathbf{x}, \tag{15}$$

and the unconditional covariance of excess payoffs is given by:

$$Cov\left[\Theta - R^f F^{-1}\mathbf{p}\right] = \tau_\theta^{-1} Id_N + \gamma^2\tau_\theta^{-2} F'\tau_x^{-1} F. \tag{16}$$

In this setting, asset returns are correlated only insofar as their payoffs load on the common asset fundamental θ_1. When there are fund managers who acquire private information, however, returns become further correlated because conditional beliefs about asset fundamentals become correlated ex-post through learning from prices. We summarize this feature of our multi-asset noisy rational expectations equilibrium in Proposition 4.

Proposition 4 *The tracking error in the market's beliefs about the aggregate fundamental θ_1 is related to its tracking error in the asset-specific fundamental $\theta_i, i \in [2, 3, ...N]$ through:*

$$\theta_1 - \hat{\theta}_1 = \sum_{i=2}^{N} b_i\left(\theta_i - \hat{\theta}_i\right) + \frac{\gamma_M \rho_S}{\chi}\left(M + e_{1j}\right)^{-1}\left(x_1 - \hat{x}_1\right),$$

where x_1 is the liquidity shock to the asset whose payoff loads only on aggregate risk. It follows that correlation between the asset-specific risky payoff of asset $i \in [2, 3, ...N]$, θ_i, and *the aggregate fundamental* θ_1 *depends on* b_i*, such that* $b_i Cov(\theta_i, \theta_1 \mid \mathcal{F}^c) \geq 0$*. The correlation between the asset-specific risky payoffs of assets* i *and* j*,* θ_i *and* θ_j*, respectively, depend on their* $b's$*, such that* $b_i b_j Cov(\theta_i, \theta_j \mid \mathcal{F}^c) \leq 0$*.*

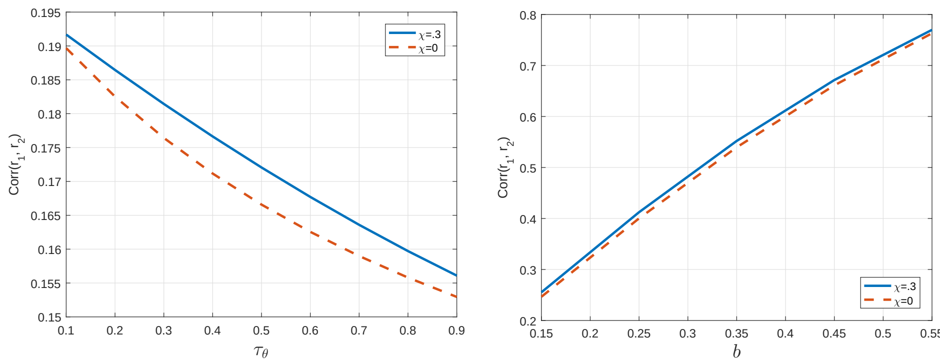
Admati (1985) emphasizes that, in a multi-asset setting, asset fundamentals become correlated through the learning channel once investors observe prices. Our setting allows us to refine this insight by highlighting the role that the correlation structure of asset payoffs plays in shaping investor expectations. As a result of the common component of asset payoff risk θ_1, the asset-specific fundamentals $\theta_i, i \in [2, 3, ...N]$ are ex-post correlated with each other after managers observe prices, and the sign of this correlation depends on the exposure of each asset to θ_1. If payoffs of assets i and j are positively correlated with θ_1, $b_i b_j > 0$, then these two asset specific shocks are negatively correlated with each other through learning, since observing two higher-than-expected prices, P_i and P_j, leads investors to revise their expectations of θ_1 upwards, and their expectations of θ_i and θ_j downwards. A similar intuition applies to the correlation between θ_i and θ_1. If $b_i > 0$, then a higher-than-expected price P_i leads investors to attribute the positive surprise to both a higher θ_i and a higher θ_1. In this sense, prices act as additional signals about unrelated asset-specific fundamentals through their common dependence on θ_1.[24] In contrast to Admati (1985), this induced correlation structure through learning also feeds back into the information acquisition decision of fund managers, and their performance-based incentives.

This induced correlation structure from learning allows us to understand the comovement of asset prices. Pindyck and Rotemberg (1990), Pindyck and Rotemberg (1993), and Barberis et al. (2005) find that asset returns appear to comove in excess of the correlation implied by their dependence on common fundamentals. Veldkamp (2006) rationalizes this phenomena in a learning framework in which information markets induce complementarities in learning about a subset of the fundamentals that drive asset prices. Similar to Veldkamp (2006), we view the limit of no information in our setting, $\chi \searrow 0$, as a benchmark for the correlation

[24] We expect that these correlations for θ_i are increasing in asset $i's$ loading b_i on the aggregate payoff fundamental θ_1.

Figure 7: **Excess Comovement: Three Assets Case**

This plot compares the correlation of asset returns when fund managers learn from private signals with the correlation of asset returns under no-information benchmark. We consider a three-asset environment, where the payoff of Asset 1 depends only on the aggregate fundamental θ_1, and the payoffs of Asset 2 and Asset 3 have both aggregate fundamental and the asset-specific fundamentals. The plots report the correlation of returns between Asset 1 and Asset 2. The Parameter: $\gamma = 1$, $\gamma_M = 2$, $R^f = 1.02$, $N = 3$, $\overline{\Theta} = .5$, $\chi = .3$, $\tau_x = 1$.



(a) Corr (Asset 1, Asset 2): τ_θ (b) Corr (Asset 1, Asset 2): b

that an econometrician would expect when estimating a model of asset returns given their common fundamentals. We also frame excess comovement in terms of excess correlation, since varying parameters related to uncertainty in the model will affect the overall level of risk in the market, and therefore the variances of asset prices.

As Panel (a) of Figure 7 illustrates, the correlation between the returns of Assets 1 and 2 can be higher than in the no information benchmark ($\chi = 0$) when uncertainty, measured by the inverse of the precision of the common prior about fundamentals τ_θ^{-1}, is sufficiently high. For low τ_θ, fund managers have a strong incentive to acquire information about the fundamentals, which become ex-post correlated in beliefs after observing prices. The impact of this correlation structure on asset prices through their trading can cause asset returns to have a higher correlation than that implied by the correlation of their payoffs. As such, a prediction of our model is that excess comovement among assets is likely to be higher in periods of high aggregate uncertainty or among assets with higher uncertainty.

When altering the exogenous correlation of asset payoffs b in Panel (b), the learning channel also generates reasonable amount of excess correlation. This occurs because the correlation structure of asset returns impacts the information that fund managers choose to acquire, and this feeds back into prices through their portfolio decisions. Since managers learn from prices, prices act as a coordination mechanism for both their information acquisition and portfolio decisions. This can potentially amplify the correlation of asset returns beyond the fundamental correlation of their asset payoffs, which is reflected in the no information benchmark, through their trading. This suggests that generating excess comovement relies on our endogenous learning channel.

5 Extensions

In this section, we discuss three extensions of our delegated learning channel. We first analyze a setting in which fund managers can trade over multiple periods. We then investigate an extension in which investors are endowed with nontradable background risk to their wealth that is correlated with the asset fundamentals. Finally, we contrast our delegated learning channel with another that is commonly explored in the literature in which managers are endowed with heterogeneous skill, and investors learn about the skill of their manager over time.

5.1 Trading over Multiple Periods

In this section, we discuss a dynamic extension of our model in which fund managers trade over multiple periods. Further details of the model, its derivation, and a more thorough discussion of the results are in the Internet Appendix A. In what follows, we describe the salient features of the analysis.

We use the multi-period model to generalize several of our insights to a dynamic setting. As is endemic to dynamic portfolio choice problems, the portfolio allocation decisions of managers and direct investors not only reflect a speculative component based on the mean and variances of asset payoffs, but also an intertemporal hedging motive to insure against future fluctuations in the payoff environment. Novel to our setting is that the effort choices of fund managers are now also forward-looking. Whereas in the static setting, fund managers seek to minimize the conditional variance of their portfolio excess payoff through their information acquisition decision, in the dynamic setting managers take into account the benefits of learning in early versus later periods for the same level of disutility from exerting effort. Fund investors take into account these intertemporal incentives when choosing the optimal affine contract to offer to fund managers.

An important point of departure of the multi-period model from its static counterpart is the possibility that investors can observe a time series of fund manager performance during the intermediate trading periods. In practice, the SEC N-Q and N-CSR filings, which are publicly available, require large mutual funds to report all long positions held at the end of a quarter. This potentially enables investors to improve their monitoring of fund managers' behavior. Based on the availability of the SEC N-Q and N-CSR filings for mutual funds, we allow investors to observe an unbiased but noisy measure of their fund's return gap at each date t. This noise, which we assume is i.i.d. across assets and dates, can be thought of as portfolio rebalancing driven by non-fundamental, non-informational reasons or measurement error. Let R_t^i be this noisy observation. Given their observations of past asset and fund-specific returns, investors can form their posterior beliefs about their fund manager's private information conditional on a given path of effort $\{\mathbf{e}_t\,(i)\}_{s=1}^{T}$. They can then derive a log-likelihood ratio under the null hypothesis that their manager exerts no effort $H_0 : \{\mathbf{e}_t\,(i)\}_{s=1}^{T} = \{\mathbf{0}_{N\times 1}\}_{s=1}^{T}$, which corresponds to no exhibition of ability. We show that the weighted historical variance of this return gap S_T is related to the log-likelihood ratio of the null hypothesis of no effort to an alternative hypothesis of some schedule of effort:

$$S_T = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{\left(R_t^i\right)^2}{Var_U\left[R_t^i \mid \mathcal{F}_t^c\right]} - \frac{\left(R_t^i - E\left[R_t^i \mid \mathcal{F}_t^I\,(i)\right]\right)^2}{Var\left[R_t^i \mid \mathcal{F}_t^I\,(i)\right]}\right),$$

and that it is a consistent estimator of whether or not the fund manager exhibits skills. While Admati and Pfleiderer (1997) demonstrate that the first moment of benchmark-adjusted returns is not sufficient to identify skill, our analysis suggests that, instead, investors should focus on second moments when evaluating the skill of fund managers. In the above statistic, $Var_U\left[R_t^i \mid \mathcal{F}_t^c\right]$ is the conditional variance of the return gap if the manager exerts no effort to learn or has no skill, $E\left[R_t^i \mid \mathcal{F}_{t-1}^I(i)\right]$ is the best predictor of the return gap given all public information available to investors, including current realized asset returns, and past fund returns, and $Var\left[R_t^i \mid \mathcal{F}_t^I(i)\right]$ is the corresponding conditional variance.

If one believes that the value that fund managers add to asset management is their acquisition of superior private information, then one should examine the variance of their return gap over time, especially since first moments are highly path-dependent. Intuitively, in any given trading period, fund managers may appear more or less active because of noise in their information or time variation in expected returns: however, systematically their portfolios should deviate from their more passive counterparts. This motivates examining dynamic measures of skill, such as the empirical analogue of our S_T statistic, to measure fund manager skill in an active management.

While the S_T statistic would allow for better monitoring of fund managers, since it could relax their IC constraints, such asymmetric payoff provisions are prohibited by the SEC. Though backward-looking as a measure, if investors allocate capital to funds based on the historical S_T statistic, then managers would face forward-looking incentives for effort to achieve a higher S_T statistic, and through this channel it would dynamically enter their compensation. To see this, suppose that a new generation of investors at date T cannot distinguish between delegating investors and direct investors, and allocates their capital to funds in proportion to the signal that the fund manager has skill based on the fund's returns, according to the rule $w(S_T)$:

$$w(S_T) = (S_T - S_{crit,\alpha})\, W_0,$$

where $\frac{S_{crit,\alpha}}{\sigma_0} = \Phi^{-1}(1-\alpha)$ is the $\alpha-$level of confidence under the null hypothesis, and $\Phi(\cdot)$ is the CDF of the normal distribution. [25] Importantly, $w(S_T)$ is increasing and convex in the most recent fund return gap R_T^i through S_T, leading to a short-term convex flow-performance relation. In addition, if the compensation of fund managers is based on their final AUM at date T, so that:

$$C_T^S = \rho_0 + \rho_S \left(W_T^S - \left(R^f\right)^T W_0 + w(S_T) \right) + \rho_R' \sum_{t=1}^{T} \mathbf{R}_t,$$

which is still a symmetric performance contract, then the future flow-performance sensitivity can incentivize forward-looking fund managers to exert effort before date t to raise their fund flows at date T. Such a mechanism suggests that convex flow-performance sensitivity may be a reaction to information about manager skill through this S_T statistic, and nonlinear flow-performance sensitivity could be a tool for completing the contracting space that is restricted in direct compensation by the SEC to linear contracts.

5.2 Background Risk

In this section, we describe an extension of the model in which investors are endowed with some exposure ψ to wealth correlated with the excess return of the asset fundamentals that they cannot trade. This specification is meant to capture the idea that investors face background risk that is correlated with asset market fluctuations.[26] Specifically, we assume that each investor's final wealth is now given by:

$$w_2^I = W_2^i - C_0^i + \psi \left(\Theta - R^f F^{-1} \mathbf{P} + \varepsilon_I \right),$$

[25] We assume implicitly that T is large enough that the asymptotic distribution is a reasonable approximation. Under the null hypothesis of no skill, or $R_t^i \sim iid\left(0, Var_U\left[R_t^i \mid \mathcal{F}_t^c\right]\right)$, S_T has a normal asymptotic distribution $\mathcal{N}\left(0, \sigma_0^2\right)$, where σ_0^2 is the variance of S_T when R_t^i is i.i.d.

[26] The assumption that investors are endowed with risk correlated with the excess return to the fundamentals $\Theta - R^f F^{-1}\mathbf{P}$ rather than the fundamentals directly is only for expositional convenience.

where $\varepsilon_I \sim \mathcal{N}\left(\mathbf{0}, \tau_I^{-1} Id_N\right)$ is independent of Θ and $\mathbf{x}$, and across investors. One can interpret ε_I as income risk that is specific to the investor or as idiosyncratic asset payoffs from market incompleteness. For brevity, we only highlight the salient features that distinguish the equilibrium from that in our baseline model, with further details available in the Internet Appendix B.

The aggregate background risk of investors shifts equilibrium asset prices by a factor $-\frac{1}{R^f} F\left(\frac{\chi}{\gamma_M \rho_S}\Omega\left(j\right)^{-1} + \frac{1-\chi}{\gamma}\Omega^{-1}\right)^{-1}\psi$, which reflects the aggregate hedging demand of investors, and modifies the benchmark for fund managers from ω^0 to ω^ψ

$$\omega^\psi = \omega^0 - \frac{\rho_S}{\rho_S + \frac{\gamma}{\gamma_M}\left(1-\rho_S\right)} F'^{-1}\psi',$$

While direct investors will incorporate their background risk into their portfolio choice, incentives must be offered to fund managers to internalize this risk by choosing the appropriate benchmark. This adjusted benchmark may also potentially impact the optimal sensitivity of the fund managers' performance-based compensation, ρ_S, and, through this delegated learning channel, their incentives to acquire private information.[27]

Savov (2014) argues that active management can provide superior insurance to investors against their aggregate income risk. Our analysis suggests that, in the presence of moral hazards in delegation, fund managers internalize such risks through the benchmark against which they are evaluated. Investors in our setting choose funds with benchmarks that reflect their hedging needs. Our framework, consequently, offers insight into differences in benchmarks across funds that invest in similar asset classes, as well as an additional dimension by which the choice of benchmark adds value to investors. Investors in our setting choose funds with benchmarks that reflect their hedging needs.

[27]This can be seen from the dependence of the FONC (A5) on ρ_R and $\bar{\mathbf{Z}}$.

5.3 Learning about Assets vs. Learning about Fund Managers

Our framework explores how the skill of fund managers can be endogenous to both the economic environment and their compensation contracts. In contrast to this mechanism, a literature, which includes Berk and Green (2004), Pástor and Stambaugh (2012), Berk and van Binsbergen (2015), Barber et al. (2016), and Starks and Sun (2016), investigates how investors try to infer the persistent skill of active managers from past performance, given that managers can influence this perception. Managers in these settings have incentives to signal their skill because fund flows are sensitive to past performance, and their compensation is tied to their fund's AUM. As such, we view both channels as complementary to understanding delegated asset management.

In practice, it is difficult to disentangle a fund manager's intrinsic ability from the assets in which it invests and the incentives that it faces. A manager whose compensation is not performance-based, for instance, has little incentive to exert costly effort to provide investors with superior performance even if it is capable. Similarly, a manager who invests in assets with more volatile payoffs has more incentive to exert effort to gain an informational advantage over other managers. While the capacity to garner superior returns may be a trait inherent to active managers, the two aforementioned situations demonstrate that fund manager skill, to some extent, must be an endogenous decision. Though studies such as Starks and Sun (2016) do allow for the ability of fund managers to vary with the investment environment, our analysis suggests it is important to take into account that variation in stock-picking and market-timing abilities may also reflect an optimal response to this change in asset environment, and to changes in the incentives provided through compensation contracts.

In addition, while asymmetric information about assets and about fund manager ability provides incentives for managers and investors to learn, respectively, their incentives to learn are likely very different. Active managers can fully exploit their private information in their investment decisions, while investors can only choose whether to invest in a fund.

This limited ability for investors to act on negative news about managers suggests that the expected return from exerting costly resources to learn is likely to be higher for managers than for investors. In addition, since private information is imperfect, measures of skill derived from a short history of the level of fund returns are likely to be noisy predictors of future performance. As such, the learning process is likely to be slower for investors and more dependent on publicly available information, such as realized past performance.

6 Conclusion

We study an economy in which investors delegate their investment in financial markets to fund managers, and must incentivize fund managers to exert costly effort to acquire private information about asset payoffs. This framework features an optimal affine contract that has both performance-based and benchmarking incentives, and allows for the study of the interaction between a manager's incentives and its learning and trading decisions. We offer novel predictions about intermediary asset holdings and asset prices that condition on the asset environment rather than the contract or managers' beliefs, which are easier to test empirically. Our model cautions the use of existing empirical measures of skill employed in the literature and offers a new measure theoretically motivated by a dynamic extension of our framework.

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A Appendix

A.1 Equilibrium Asset Prices

Given the asset demand of direct investors and fund managers from equation (9) and Proposition 1, respectively, we are now in a position to derive equilibrium prices. Aggregating the demand of fund managers and direct investors, $\omega_1^S(j)$ and ω_1^D, respectively, the market-clearing condition reveals that:

$$\chi \frac{1}{\gamma_M \rho_S} \left(F\Omega(j) F'\right)^{-1} \left(F \int_0^1 \hat{\Theta}(j)\, di - R^f \mathbf{P}\right) - \chi \frac{1}{\rho_S} \rho_{\mathbf{R}} + (1-\chi) \frac{1}{\gamma} \left(F\Omega F'\right)^{-1} \left(F\hat{\Theta} - R^f \mathbf{P}\right) = \mathbf{x}.$$

Substituting for $\hat{\Theta}$ and $\hat{\Theta}(j)$ with equations (5) and (7), respectively, and imposing the Strong LLN, we find that:

$$\begin{aligned} \mathbf{P} &= \left(\left(\frac{\chi}{\gamma_M \rho_S}\Omega(j)^{-1} + \frac{1-\chi}{\gamma}\Omega^{-1}\right) R^f F^{-1} - \left(\frac{1-\chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S}\right) \tau_x \Pi_\theta' \left(\Pi_x \Pi_x'\right)^{-1}\right)^{-1} \times \\ &\quad \left(\left(\frac{1-\chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S}\right) \left(\tau_\theta \bar{\Theta} - \tau_x \Pi_\theta' \left(\Pi_x \Pi_x'\right)^{-1} (\Pi_0 + \Pi_x \bar{\mathbf{x}})\right) + \frac{\chi}{\gamma_M \rho_S} \Sigma_j (\mathbf{e}_j)^{-1} \Theta - F'\mathbf{x} - \frac{\chi}{\rho_S} F' \rho_{\mathbf{R}}\right). \end{aligned}$$

Matching coefficients with the conjectured form of prices (4), and the imposing equation (6), we find that:

$$\Omega^{-1} = \tau_\theta Id_N + \tau_x \left(\frac{\chi}{\gamma_M \rho_S}\right)^2 \Sigma_j (\mathbf{e}_j)^{-1} \left(F'F\right)^{-1} \Sigma_j (\mathbf{e}_j)^{-1}, \tag{A1}$$

and that Π_θ, Π_x, and Π_0 are given by:

$$\Pi_\theta = \frac{1}{R^f} F \begin{pmatrix} \tau_\theta \left(\tau_x \left(\frac{\chi}{\gamma_M \rho_S}\right)^2 \Sigma_j (\mathbf{e}_j)^{-1} (F'F)^{-1} \Sigma_j (\mathbf{e}_j)^{-1} + \left(1 + \frac{1-\chi}{\chi}\frac{\gamma_M}{\gamma}\rho_S\right)^{-1} \Sigma_j (\mathbf{e}_j)^{-1}\right)^{-1} \\ + Id_N \end{pmatrix}^{-1} \tag{A2}$$

$$\Pi_x = -\frac{\gamma_M \rho_S}{\chi} \Pi_\theta \Sigma_j (\mathbf{e}_j) F' \tag{A3}$$

$$\Pi_0 = \frac{1}{R^f} F \left(\frac{\chi}{\gamma_M \rho_S}\Omega(j)^{-1} + \frac{1-\chi}{\gamma}\Omega^{-1}\right)^{-1} \begin{pmatrix} \left(\frac{1-\chi}{\gamma} + \frac{\chi}{\gamma_M \rho_S}\right) \left(\tau_\theta \bar{\theta} - \tau_x \Pi_\theta' \Pi_x'^{-1} \bar{\mathbf{x}}\right) \\ -\frac{\chi}{\rho_S} F' \rho_{\mathbf{R}} \end{pmatrix}, \tag{A4}$$

which confirms the conjectured linear equilibrium.

Several features of the equilibrium are immediately apparent from the price coefficients.

We see, for instance, that if $\Sigma_j (\mathbf{e}_j)^{-1}$ is zero, so that fund managers have no private information, then $\Pi_\theta, \Pi_x \to 0_{N\times N}$, and prices reflect only prior information about the risky asset payoffs. In addition, the signal-to-noise ratio of prices as signals about the risky asset payoffs, $\Pi_x^{-1}\Pi_\theta = -\frac{\chi}{\gamma_M \rho_S} F'^{-1}\Sigma_j (\mathbf{e}_j)^{-1}$, depends not only on the correlation structure of asset payoffs and the effort exerted by fund managers, but also negatively on their risk aversion γ_M and the sensitivity of their compensation to the realized return of their fund, ρ_S. That these latter two features enter as $\gamma_M \rho_S$ highlights that ρ_S makes the fund manager effectively more risk-averse over his fund's performance, and, as a result, more conservative in his investment policies.

A.2 Expected Utility of Direct Investors

We calculate the expected utility of direct investors V_0^D. By the law of iterated expectations, first conditioning on $\mathcal{F}^c$, the expected utility to direct investors V_0^D is then:

$$V_0^D = -E\left[\exp\left(-\gamma\left(W_2^D\right)\right)\right] = -E\left[\exp\left(-\gamma R^f W_0 - \frac{1}{2}\mathbf{Z}'\Omega^{-1}\mathbf{Z}\right)\right],$$

where $\mathbf{Z} = \hat{\Theta} - R^f F^{-1}\mathbf{P}$. From an ex-ante perspective, $\mathbf{Z} \sim \mathcal{N}(\mu, \Omega_Z)$. With some manipulation,

$$\begin{aligned} \mu &= \bar{\Theta} - R^f F^{-1}\bar{\mathbf{P}}, \\ \Omega_Z &= \left(\Omega^{-1} + \left(1 + \frac{1-\chi}{\chi}\frac{\gamma_M}{\gamma}\rho_S\right)\Omega^{-1}\Sigma_j(\mathbf{e}_j)\,\Omega^{-1}\right)^{-1} \times \\ &\quad \left(\tau_\Theta^{-1} Id_N - \Omega\right)^{-1}\left(\Omega^{-1} + \left(1 + \frac{1-\chi}{\chi}\frac{\gamma_M}{\gamma}\rho_S\right)\Omega^{-1}\Sigma_j(\mathbf{e}_j)\,\Omega^{-1}\right)^{-1}, \end{aligned}$$

and $\bar{\mathbf{P}} = \Pi_0 + \Pi_\theta\bar{\Theta} + \Pi_x\bar{\mathbf{x}}$. Then, by competing the square,

$$V_0^D = -\frac{\exp\left(-\gamma R^f W_0 - \frac{1}{2}\mu'\left(\Omega_Z^{-1} - \Omega_Z^{-1}\left(\Omega^{-1} + \Omega_Z^{-1}\right)^{-1}\Omega_Z^{-1}\right)\mu\right)}{\left|Id_N + \Omega_Z\Omega^{-1}\right|}.$$

A.3 Proof of Proposition 1

Assuming the linear contract, the IC constraint of the fund manager, conditional on an effort choice $\mathbf{e}$, reduces to the mean-variance optimization problem:

$$\sup_{\omega_1^S(j)} \left\{ \begin{array}{c} \rho_0 + \rho_S\omega_1^S(j)'\left(F\hat{\Theta}(j) - R^f\mathbf{P}\right) + \rho_{\mathbf{R}}'\left(F\hat{\Theta}(j) - R^f\mathbf{P}\right) \\ -\frac{\gamma_M}{2}\left(\rho_S\omega_1^S(j) + \rho_{\mathbf{R}}\right)' F\Omega(j)\, F'\left(\rho_S\omega_1^S(j) + \rho_{\mathbf{R}}\right) \end{array} \right\},$$

given its CARA-normal structure. It then follows from the FOC for ω_1^S at interior solution that:

$$\omega_1^S(j) = \frac{1}{\gamma_M \rho_S}\left(F\Omega(j)F'\right)^{-1}\left(F\hat{\Theta}(j) - R^f\mathbf{P}\right) - \frac{1}{\rho_S}\rho_{\mathbf{R}}.$$

Substituting this optimal portfolio choice into the manager's utility, the IC constraint when choosing effort level $\mathbf{e}$ becomes:

$$\mathbf{e} \in \text{argsup}_{\mathbf{e}_j \in \mathbb{R}_+^N}\left\{E\left[-\exp\left(\begin{array}{c} H(\mathbf{e}_j) - \gamma_M\rho_0 \\ -\frac{1}{2}\left(\hat{\Theta}(j) - R^f F^{-1}\mathbf{P}\right)'\Omega(j)^{-1}\left(\hat{\Theta}(j) - R^f F^{-1}\mathbf{P}\right)\end{array}\right)\right]\right\}.$$

To solve for the optimal level of effort for fund managers, we invoke the law of iterated expectations and first find the expected utility of a fund manager conditional on having observed market prices. The optimal choice of effort conditional on having observed market prices is independent of the specific realization of prices. As a result, the optimal effort of fund managers conditional on observing prices is also a measurable strategy for fund managers before observing prices. Since unconditional strategies cannot improve on strategies that condition on more information, this optimal effort ex-post must also be optimal ex-ante.

Recognizing that $\mathbf{s}(j) \mid \mathcal{F}_0^c \sim \mathcal{N}\left(\hat{\Theta}, \Omega + \Sigma_j(\mathbf{e}_j)\right)$, and that

$$\hat{\Theta}(j) - R^f F^{-1}\mathbf{P} = \hat{\Theta} - R^f F^{-1}\mathbf{P} + \Omega(j)\Sigma_j(\mathbf{e}_j)^{-1}\left(\mathbf{s}(j) - \hat{\Theta}\right),$$

where

$$\Omega(j)^{-1} = \Omega^{-1} + \Sigma_j(\mathbf{e}_j)^{-1},$$

by completing the square for normal random variables, the expected utility of fund manager i given only the market beliefs and effort $\mathbf{e}'$ $E\left[\sup_{\omega\in\mathbb{R}\mathbb{R}^N} E\left[u\left(C_0^S;\omega',\mathbf{e}'\right) \mid \mathcal{F}_j\right] \mid \mathcal{F}^c\right]$ is

$$\begin{aligned}
& E\left[\sup_{\omega\in\mathbb{R}^N} E\left[u\left(C_0^S;\omega',\mathbf{e}'\right) \mid \mathcal{F}_j\right] \mid \mathcal{F}_0^c\right] \\
= \quad & -E\left[\exp\left(\begin{array}{c}\frac{1}{2}h\left((\mathbf{e}')'\mathbf{1}_{N\times 1}\right) - \gamma_M\rho_0 \\ -\frac{1}{2}\left(\hat{\Theta}(j) - R^f F^{-1}\mathbf{P}\right)'\Omega(j)^{-1}\left(\hat{\Theta}(j) - R^f F^{-1}\mathbf{P}\right)\end{array}\right) \mid \mathcal{F}_0^c\right] \\
= \quad & -\frac{(2\pi)^{-\frac{N}{2}}}{|\Omega + \Sigma_j(\mathbf{e}_j)|^{1/2}}\int_{-\infty}^{\infty}\exp\left(\begin{array}{c}\frac{1}{2}h\left((\mathbf{e}')'\mathbf{1}_{N\times 1}\right) - \gamma_M\rho_0 \\ -\frac{1}{2}\left(\hat{\Theta}(j) - R^f F^{-1}\mathbf{P}\right)'\Omega(j)^{-1}\left(\hat{\Theta}(j) - R^f F^{-1}\mathbf{P}\right) \\ -\frac{1}{2}\left(\mathbf{s}(j) - \hat{\Theta}\right)'\left(\Omega + \Sigma_j(\mathbf{e}_j)\right)^{-1}\left(\mathbf{s}(j) - \hat{\Theta}\right)\end{array}\right) d\mathbf{s}(j)
\end{aligned}$$

$$= \quad -\frac{(2\pi)^{-\frac{N}{2}}}{\left|\Omega+\Sigma_j(\mathbf{e}_j)\right|^{1/2}}\int_{-\infty}^{\infty}\exp\left(\begin{array}{c}\frac{1}{2}h\left((\mathbf{e}')'\mathbf{1}_{N\times1}\right)-\gamma_M\rho_0\\ -\frac{1}{2}\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}\right)'\Omega^{-1}\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}\right)\\ -\frac{1}{2}\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}+\mathbf{s}(j)-\hat{\Theta}\right)'\Sigma_j(\mathbf{e}_j)^{-1}\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}+\mathbf{s}(j)-\hat{\Theta}\right)\end{array}\right)d\mathbf{s}(j)$$

$$= \quad -\exp\left(\begin{array}{c}\frac{1}{2}h\left((\mathbf{e}')'\mathbf{1}_{N\times1}\right)-\frac{1}{2}\log\left|Id_N+\Sigma_j(\mathbf{e}')^{-1}\Omega\right|-\gamma_M\rho_0\\ -\frac{1}{2}\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}\right)'\Omega^{-1}\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}\right)\end{array}\right).$$

A similar result can be found by applying results for the moment-generating function of the non-central chi-square random variables. As one can see, the optimal choice of effort enters the conditional expected utility only through the $-\frac{1}{2}\log\left|Id_N+\Sigma_j(\mathbf{e}_j)^{-1}\Omega\right|$ term. Since fund managers are price-takers and the conditional variance of market beliefs Ω is known ex-ante, we find that:

$$\begin{aligned}E\left[\sup_{\omega\in\mathbb{R}^N}E\left[u\left(C_0^S;\omega',\mathbf{e}'\right)\right]\right] &= -\exp\left(\frac{1}{2}h\left((\mathbf{e}')'\mathbf{1}_{N\times1}\right)-\frac{1}{2}\log\left|Id_N+\Sigma_j(\mathbf{e}')^{-1}\Omega\right|-\gamma_M\rho_0\right)\\ &\quad\times E\left[\exp\left(-\frac{1}{2}\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}\right)'\Omega^{-1}\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}\right)\right)\right].\end{aligned}$$

The optimization program for the effort of fund manager is then equivalent to:

$$\mathbf{e}\in\operatorname{argsup}_{\mathbf{e}'\in\mathbb{R}_+^N}\left\{\log\left|\Omega^{-1}+\Sigma_j(\mathbf{e}')^{-1}\right|-h\left((\mathbf{e}')'\mathbf{1}_{N\times1}\right)\right\}.$$

Recognizing that $\Sigma_j(\mathbf{e}_j)^{-1}=M\cdot Id_N+diag(\mathbf{e})$, and invoking results of the matrix calculus, the FOC for the optimal level of effort $\mathbf{e}_i$ is:

$$Tr\left[\left(\Omega^{-1}+M\cdot Id_N+diag(\mathbf{e})\right)^{-1}J_i\right]-h'\left(\mathbf{e}'\mathbf{1}_{N\times1}\right)\le 0\ \ (=\ \ if\ \ e_i>0).$$

where J_i is the $N\times N$ matrix with entry $J_{ii}=1$ and zero otherwise. Since Tr is a linear operator, we can stack all the FOCs to arrive at:

$$Diag\left[\left(\Omega^{-1}+M\cdot Id_N+diag(\mathbf{e})\right)^{-1}\right]-h'\left(\mathbf{e}'\mathbf{1}_{N\times1}\right)\mathbf{1}_{N\times1}\le\mathbf{0}_{N\times1},$$

where $Diag$ is the operator that stacks the diagonal of a matrix into a vector. Furthermore, the second-order derivative of $\log\left|\Omega^{-1}+\Sigma_j(\mathbf{e}')^{-1}\right|$ is:

$$\partial^2_{\mathbf{e}_i\mathbf{e}_i}\log\left|\Omega^{-1}+\Sigma_j(\mathbf{e}')^{-1}\right|=-\left(\Omega^{-1}+M\cdot Id_N+diag(\mathbf{e})\right)^{-1}J_i\left(\Omega^{-1}+M\cdot Id_N+diag(\mathbf{e})\right)^{-1}.$$

Since $h'(\cdot)$ is a (weakly) convex function, the optimization program is concave in $\mathbf{e}$, and therefore the FOC is both necessary and sufficient for the optimal $\mathbf{e}$.

If F is diagonal, so that asset payoffs are independent, then Ω^{-1} is also diagonal, and the above condition reduces to:

$$\frac{1}{\Omega_{ii}^{-1} + M + \mathbf{e}_i} \leq h'\left(\mathbf{e}'\mathbf{1}_{N\times 1}\right) \;\; \forall \; i \in \{1, ..., N\}.$$

A.4 Proof of Proposition 2

Define

$$G = Tr\left[X^{-1}J_i\right] - h'\left(\mathbf{e}'\mathbf{1}_{N\times 1}\right) = 0,$$

where $X = \left((\tau_\theta + M)\cdot Id_N + k\left(M\cdot Id_N + diag\left(\mathbf{e}\right)\right)\left(F'F\right)^{-1}\left(M\cdot Id_N + diag\left(\mathbf{e}\right)\right) + diag\left(\mathbf{e}\right)\right) J_{ii}$, and $k = \tau_x\left(\frac{\chi}{\gamma_M \rho_S}\right)^2$. By the implicit function theorem,

$$\partial_z \mathbf{e}_i = -\frac{\partial_z G}{\partial_{\mathbf{e}_i} G},$$

for parameter z. Recognizing that $\partial\left(X^{-1}\right) = X^{-1}\left(\partial X\right)X^{-1}$, taking the derivative under the Tr operator since the Tr operator is linear and the trace is bounded, it follows that:

$$\partial_{\mathbf{e}_i} G = Tr\left[AJ_i\right] = \mathbf{v}_i' A \mathbf{v}_i - h''\left(\mathbf{e}'\mathbf{1}_{N\times 1}\right),$$

where J_i is the $N\times N$ matrix with entry $J_{ii} = 1$ and zero otherwise, $\mathbf{v}_i$ is the Euclidian $N \times N$ basis vector in the i^{th} direction, and

$$\begin{aligned} A &= -X^{-1}\left(k\left(M\cdot Id_N + diag\left(\mathbf{e}\right)\right)F^{-1}F'^{-1} + k\left(F'F\right)^{-1}\left(M\cdot Id_N + diag\left(\mathbf{e}\right)\right) + Id_N\right)X^{-1} \\ &\quad -h''\left(\mathbf{e}'\mathbf{1}_{N\times 1}\right) Id_N. \end{aligned}$$

Given that F is a lower triangular matrix with entries of 1 on the diagonal, $F'F$ is a positive definite (PD) matrix since $\det(AB) = \det(A)\det(B)$. Since $F'F$ is a positive definite (PD) matrix, it follows that $\left(F'F\right)^{-1}$ is a PD matrix, since the eigenvalues of $\left(F'F\right)$ are the inverse of the eigenvalues of $F'F$. Since $\left(F'F\right)^{-1}$ is a PD matrix, X is also a PD matrix, and it follows that A is a negative definite (ND) matrix. Therefore, it follows since $\mathbf{v}_i$ has non-negative entries and $h\left(\mathbf{e}'\mathbf{1}_{N\times 1}\right)$ is convex that:

$$\partial_{\mathbf{e}_i} G = \mathbf{v}_i' A \mathbf{v}_i - h''\left(\mathbf{e}'\mathbf{1}_{N\times 1}\right) < 0.$$

Consequently,

$$\partial_z \mathbf{e}_i = \frac{\partial_z G}{\left|\partial_{\mathbf{e}_i} G\right|} = \frac{\partial_z Tr\left[X^{-1}\right]}{\left|\partial_{\mathbf{e}_i} G\right|},$$

and the sign of $\frac{\partial \mathbf{e}_i}{\partial z}$ is the same as the sign of $\partial_z Tr\left[X^{-1}\right]$. Differentiating under the Tr operator again, it follows that:

$$\partial_z G = -Tr\left[X^{-1}\partial_z\left(\left(\tau_\theta + M\right)\cdot Id_N + k\left(M\cdot Id_N + diag\left(\mathbf{e}\right)\right)\left(F'F\right)^{-1}\left(M\cdot Id_N + diag\left(\mathbf{e}\right)\right)\right)X^{-1}J_i\right].$$

For $z = \tau_\theta$, it is straightforward to verify that $\partial_{\tau_\theta} G > 0$, since $Tr\left[X^{-1}\tau_\theta X^{-1}J_i\right] = \tau_\theta Tr\left[X^{-1}X^{-1}J_i\right]$ and X is PD, and therefore:

$$\partial_{\tau_\theta}\mathbf{e}_i < 0.$$

Similarly, it follows that:

$$\partial_k \mathbf{e}_i < 0.$$

The results for elements of k then follow by the chain rule.

A.5 Proof of Proposition 3

Substituting for W_2^S and C_0^S, the utility of investors that invest with fund managers is

$$V\left(W_2^S, C_0^S\right) = -\exp\left(-\gamma\left(\begin{array}{c} R^f W_0 - \rho_0 + \frac{1-\rho_S}{\gamma_M\rho_S}\left(\mathbf{s}_j - \hat{\Theta}\right)'\Sigma_j\left(\mathbf{e}_j\right)^{-1}\left(\Theta - R^f F^{-1}\mathbf{P}\right) \\ +\frac{1}{\rho_S}\left(\frac{1-\rho_S}{\gamma_M}\left(\hat{\Theta} - R^f F^{-1}\mathbf{P}\right)'\left(\Omega^{-1} + \Sigma_j\left(\mathbf{e}_j\right)^{-1}\right) - \rho_{\mathbf{R}}'F\right)\left(\Theta - R^f F^{-1}\mathbf{P}\right)\end{array}\right)\right).$$

Importantly, $\mathbf{e}_j$ is independent of the realization of Θ. To find expected investor utility when investing with fund managers, we recognize by the law of iterated expectations that $E\left[V\left(W_2^S, C_0^S\right)\right] = E\left[E\left[V\left(W_2^S, C_0^S\right) \mid \mathcal{F}^c\right]\right]$, and that $E\left[V\left(W_2^S, C_0^S\right) \mid \mathcal{F}^c\right] = E\left[E\left[V\left(W_2^S, C_0^S\right) \mid \Theta, x\right.\right.$ Taking conditional expectations with respect to the realized shocks, and integrating over the idiosyncratic signal noise of fund managers, we find:

$$\begin{aligned} & E\left[V\left(W_2^S, C_0^S\right) \mid \Theta, x\right] \\ = \; & -\exp\left(-\gamma\left(\begin{array}{c} R^f W_0 - \rho_0 + \left(\frac{1}{2}\left(\frac{1-\rho_S}{\gamma_M\rho_S}\right)^2 + \frac{1-\rho_S}{\gamma_M\rho_S}\right)\left(\Theta - \hat{\Theta}\right)'\Sigma_j\left(\mathbf{e}_j\right)^{-1}\left(\Theta - R^f F^{-1}\mathbf{P}\right) \\ +\frac{1}{\rho_S}\left(\frac{1-\rho_S}{\gamma_M}\left(\hat{\Theta} - R^f F^{-1}\mathbf{P}\right)'\left(\Omega^{-1} + \left(1 + \frac{1}{2}\frac{1-\rho_S}{\gamma_M\rho_S}\right)\Sigma_j\left(\mathbf{e}_j\right)^{-1}\right) - \rho_{\mathbf{R}}'F\right)\left(\Theta - R^f F^{-1}\mathbf{P}\right)\end{array}\right)\right). \end{aligned}$$

Taking conditional expectations with respect to the market beliefs, we then arrive at:

$$\begin{aligned} & E\left[V\left(W_2^S, C_0^S\right) \mid \mathcal{F}^c\right] \\ = & -\frac{\exp\left(\begin{array}{c} \gamma\rho_0 - \gamma R^f W_0 - \frac{1}{2}\mathbf{Z}'\Omega^{-1}\mathbf{Z} \\ +\frac{1}{2}\left(\left(1+\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)\Omega^{-1}\mathbf{Z} - \frac{\gamma}{\rho_S}F'\rho_{\mathbf{R}}\right)'\left(\Omega^{-1}+\left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)\Sigma_j\left(\mathbf{e}_j\right)^{-1}\right)^{-1} \times \\ \left(\left(1+\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)\Omega^{-1}\mathbf{Z} - \frac{\gamma}{\rho_S}F'\rho_{\mathbf{R}}\right) \end{array}\right)}{\left|Id_N + \left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)\Omega\Sigma_j\left(\mathbf{e}_j\right)^{-1}\right|^{1/2}}, \end{aligned}$$

where $\mathbf{Z} = \hat{\Theta} - R^f F^{-1}\mathbf{P}$. From an ex ante perspective, $\mathbf{Z} \sim \mathcal{N}\left(\mu, \Omega_Z\right)$. Taking unconditional expectations, we arrive at:

$$\begin{aligned} & E\left[V\left(W_2^S, C_0^S\right)\right] \\ = & -\frac{\exp\left(\begin{array}{c} -\gamma R^f W_0 + \frac{1}{2}\left(\frac{\gamma}{\rho_S}\right)^2 \rho_{\mathbf{R}}' F\left(\Omega^{-1}+\left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)\Sigma_j\left(\mathbf{e}_j\right)^{-1}\right)^{-1} F'\rho_{\mathbf{R}} \\ +\frac{1}{2}G'H^{-1}G - \frac{1}{2}\mu'\Omega_Z^{-1}\mu + \gamma\rho_0 \end{array}\right)}{\left|Id_N + \left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)\Omega\Sigma_j\left(\mathbf{e}_j\right)^{-1}\right|^{1/2}\left|\Omega_Z H\right|^{1/2}}, \end{aligned}$$

where

$$\begin{aligned} G &= \frac{\gamma}{\rho_S}\left(1+\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)\left(Id_N + \left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)\Sigma_j\left(\mathbf{e}_j\right)^{-1}\Omega\right)^{-1} F'\rho_{\mathbf{R}} + \Omega_Z^{-1}\mu, \\ H &= \Omega^{-1} + \Omega_Z^{-1} - \left(1+\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2 \Omega^{-1}\left(\Omega^{-1}+\left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)\Sigma_j\left(\mathbf{e}_j\right)^{-1}\right)^{-1}\Omega^{-1}. \end{aligned}$$

Investors in fund managers are used to solve the optimization problem:

$$\begin{aligned} V_0^S &= \sup_{\{\rho_0,\rho_S,\rho_{\mathbf{R}}\}} E\left[V\left(W_2^S, C_0^S\right)\right] \\ s.t. &: E\left[V\left(W_2^S, C_0^S\right)\right] = V_0^D \text{ (indifference)}, \\ &: Tr\left[\left(\Omega^{-1} + M \cdot Id_N + diag\left(\mathbf{e}_j\right)\right)^{-1} J_i\right] - h'\left(\mathbf{e}_j'\mathbf{1}_{N\times 1}\right) \leq 0 \;\forall\; i \in \{1,...,N\} \text{ (optimal } \mathbf{e}_j). \end{aligned}$$

Importantly, V_0^D and the FOC for the optimal choice of fund manager effort are independent of the contract from the perspective of investors.

The FOC for $\rho_{\mathbf{R}}$ can be solved explicitly for $\rho_{\mathbf{R}}$ such that:

$$\begin{aligned}\rho_{\mathbf{R}} &= -\left(\frac{\rho_S}{\gamma}+\frac{1-\rho_S}{\gamma_M}\right)F'^{-1} \\ &\times\left(\left(1+\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\Omega^{-1}H^{-1}\Omega^{-1}\left(\Omega^{-1}+\left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)\Sigma_j\left(\mathbf{e}_j\right)^{-1}\right)^{-1}+Id_N\right)^{-1} \\ &\times\Omega^{-1}H^{-1}\Omega_Z^{-1}\mu,\end{aligned}$$

which, with some manipulation, simplifies to:

$$\rho_{\mathbf{R}} = -\left(\frac{\rho_S}{\gamma}+\frac{1-\rho_S}{\gamma_M}\right)F'^{-1}\left(\Omega_Z+\Omega\right)^{-1}\mu.$$

Recognizing that we can rewrite $F'^{-1}\left(\Omega_Z+\Omega\right)^{-1}\mu$ as $\left(F'\left(\Omega_Z+\Omega\right)F\right)^{-1}F\mu$, where $F\mu$ is the unconditional expected excess return on the risky assets, it follows that $F'^{-1}\left(\Omega_Z+\Omega\right)^{-1}\mu$ is a portfolio allocation chosen before prices are observed that accounts for both the overall uncertainty of excess returns and the uncertainty given common prices Ω augmented by the uncertainty over the realization of prices Ω_Z.

Defining $\omega^0 = \frac{1}{\gamma}F'^{-1}\left(\Omega_Z+\Omega\right)^{-1}\mu$ to represent this "naive" portfolio, we can express $\rho_{\mathbf{R}}$ as:

$$\rho_{\mathbf{R}} = -\left(\rho_S+\frac{\gamma}{\gamma_M}\left(1-\rho_S\right)\right)\omega^0.$$

Furthermore, by the law of total variance:

$$\begin{aligned}Var\left(\Theta-R^fF^{-1}\mathbf{P}\right) &= E\left[Var\left(\Theta-R^fF^{-1}\mathbf{P}\mid\mathcal{F}^c\right)\right]+Var\left(E\left[\Theta-R^fF^{-1}\mathbf{P}\mid\mathcal{F}^c\right]\right) \\ &= E\left[\Omega\right]+Var\left(\hat{\Theta}-R^fF^{-1}\mathbf{P}\right) \\ &= \Omega+\Omega_Z.\end{aligned}$$

Therefore, ω^0 can be expressed as:

$$\omega^0 = \frac{1}{\gamma}F'^{-1}Var\left(\Theta-R^fF^{-1}\mathbf{P}\right)^{-1}E\left[\Theta-R^fF^{-1}\mathbf{P}\right],$$

which is the ex-ante mean-variance efficient portfolio.

Since ρ_0 impacts V_0^S only through the $e^{\gamma\rho_0}$ term, we can define v^S, where:

$$V_0^S = e^{\gamma\rho_0}v^S,$$

and v^S is independent of ρ_0 from the perspective of delegated investors. Since the expected utility of direct investors V_0^D is independent of ρ_0 from the perspective of investors, from the indifference condition it follows that:

$$\rho_0 = \frac{1}{\gamma} \log \frac{V_0^D}{v^S}.$$

Assuming the program for the investor is concave in ρ_U and ρ_S, the optimal choices of ρ_S satisfies:

$$\rho_S = \arg\sup_{\rho_S} \left\{ \begin{array}{c} -\frac{1}{2}\left(\frac{\gamma}{\rho_S}\right)^2 \rho_{\mathbf{R}}' F \left(\Omega^{-1} + \left(1 - \left(1 - \frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right) \Sigma_j \left(\mathbf{e}_j\right)^{-1}\right)^{-1} F' \rho_{\mathbf{R}} \\ +\frac{1}{2}\log\left| Id_N + \left(1 - \left(1 - \frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right) \Omega \Sigma_j \left(\mathbf{e}_j\right)^{-1} \right| + \frac{1}{2}\log|H| - \frac{1}{2} G' H^{-1} G \end{array} \right\}.$$

Applying matrix calculus, we derive the FONC for the optimal choice of ρ_S:

$$0 = \left\{ \begin{array}{c} \frac{1}{\rho_S}\left(\frac{\gamma}{\rho_S}\right)^2 \rho_{\mathbf{R}}' F A F' \rho_{\mathbf{R}} \\ -\left(1 - \frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right) \frac{\gamma}{\gamma_M} \left(\frac{\gamma}{\rho_S^2}\right)^2 Tr\left[A F' \rho_{\mathbf{R}} \rho_{\mathbf{R}}' F A \Sigma_j \left(\mathbf{e}_j\right)^{-1}\right] \\ +\frac{1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}}{1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2} \frac{\gamma}{\gamma_M \rho_S^2} \left(N - \frac{Tr\left[\left(\left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)^{-1} Id_N + \Omega \Sigma_j (\mathbf{e}_j)^{-1}\right)^{-1} Id_N\right]}{1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2} \right)' \\ +Tr\left[\frac{\gamma}{\gamma_M}\frac{1}{\rho_S^2}\left(1 + \frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right) H^{-1}\Omega^{-1} A \Omega^{-1} \left(Id_N - \left(1 - \left(\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right) \Omega \Sigma_j \left(\mathbf{e}_j\right)^{-1} A \Omega^{-1}\right)\right] \\ +\frac{\gamma\left(1+\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)}{\gamma_M \rho_S^2} Tr\left[H^{-1} G G' H^{-1} \Omega^{-1} A \Omega^{-1} \left(Id_N - \left(1 - \left(\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right) \Omega \Sigma_j \left(\mathbf{e}_j\right)^{-1} A \Omega^{-1}\right)\right] \\ +Tr\left[F' \rho_{\mathbf{R}} G' H^{-1} \left(\frac{\gamma}{\rho_S^2} \frac{1+\frac{\gamma}{\gamma_M}\frac{2-\rho_S}{\rho_S} - 2\frac{\gamma}{\gamma_M}\frac{1}{\rho_S}\frac{1-\left(\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2}{1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2}}{1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2} B + 2\frac{\gamma}{\rho_S^3}\frac{\gamma}{\gamma_M} \frac{1-\left(\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2}{\left(1-\left(1-\frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)^3} BB \right)\right] \end{array} \right\}, \tag{A5}$$

where:

$$\begin{aligned} A &= \left(\Omega^{-1} + \left(1 - \left(1 - \frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right) \Sigma_j \left(\mathbf{e}_j\right)^{-1}\right)^{-1}, \\ B &= \left(\left(1 - \left(1 - \frac{\gamma}{\gamma_M}\frac{1-\rho_S}{\rho_S}\right)^2\right)^{-1} Id_N + \Sigma_j \left(\mathbf{e}_j\right)^{-1} \Omega\right)^{-1}. \end{aligned}$$

A.6 Proof of Proposition 4

Given the functional form of the price $\mathbf{P}$, and that it is publicly observed, we can decompose it as:

$$\Pi_{\theta}^{-1}\left(\mathbf{P}-\Pi_0\right)=\Theta-\frac{\gamma_M\rho_S}{\chi}\Sigma_j\left(\mathbf{e}_j\right)F'\mathbf{x}=\hat{\Theta}-\frac{\gamma_M\rho_S}{\chi}\Sigma_j\left(\mathbf{e}_j\right)F'\hat{\mathbf{x}},$$

from which it follows that

$$\Theta-\hat{\Theta}=\frac{\gamma_M\rho_S}{\chi}\Sigma_j\left(\mathbf{e}_j\right)F'\left(\mathbf{x}-\hat{\mathbf{x}}\right), \tag{A6}$$

where we have substituted for $\Pi_{\theta}^{-1}\Pi_x$ with our expression in the main text, $\Pi_{\theta}=-\Pi_x\frac{\chi}{\gamma_M\rho_S}F'^{-1}\Sigma_j\left(\mathbf{e}_j\right)^{-1}$. It follows that beliefs across $\hat{\Theta}$ are correlated, and the innovations to the supply shocks $\hat{\mathbf{x}}-\mathbf{x}$ are entangled by the payoff matrix F'. Notice that the first row of F' is the vector of asset $b's$, and F' is the bordered identity matrix Id_{N-1} for rows 2 to N.

Substituting with the system of equations (A6), and recognizing that $K_{ii}\left(e_{ij}\right)=M+e_{ij}$, we obtain:

$$\theta_1-\hat{\theta}_1=\frac{\gamma_M\rho_S}{\chi}\sum_{i=2}^{N}K_{ii}\left(e_{ij}\right)^{-1}b_i\left(x_i-\hat{x}_i\right)=\frac{\gamma_M\rho_S}{\chi}K_{11}\left(e_{1j}\right)^{-1}\left(x_1-\hat{x}_1\right)+\sum_{i=1}^{N-1}b_i\left(\theta_i-\hat{\theta}_i\right),$$

with the understanding that $b_1=1$. It then follows that if the market overestimates θ_i, $\theta_i<\hat{\theta}_i, i\in[2,3,...N]$, that it marginally overestimates the aggregate risk θ_1 if $b_i\geq 0$, $\theta_1<\hat{\theta}_1$. Consequently, it follows that:

$$b_iCov\left(\theta_i,\theta_1\mid\mathcal{F}^c\right)\geq 0\;\forall\;i\in\{2,...,N\}.$$

Therefore, if the market overestimates the asset-specific payoff to asset i, it will also overestimate the aggregate fundamental θ_1, all else equal and all other prices held constant.

Next we fix the market's perception of the aggregate risk $\hat{\theta}_1$. A positive shift in the perception of asset-specific payoff to asset i, $\theta_i-\hat{\theta}_i, i\in[2,3,...N]$, must be offset by a shift in the perception of the remaining asset-specific asset payoffs $\sum_{2,j\neq i}^{N}\frac{b_j}{b_i}\left(\theta_j-\hat{\theta}_j\right)$ to hold fixed the perception of aggregate risk $\hat{\theta}_1$. It then follows that:

$$b_ib_jCov\left(\hat{\theta}_i,\hat{\theta}_j\mid\mathcal{F}^c\right)\leq 0\;\forall\;\left(i,j\right)\;i\neq j.$$