Foundation for the Advancement of Research in Financial Economics

Transform Analysis and Asset Pricing for Affine Jump-Diffusions
by Darrell Duffie, Jun Pan, Kenneth Singleton

The fourth Stephen A. Ross Prize in Financial Economics has been awarded to "Transform Analysis and Asset Pricing for Affine Jump-Diffusions" published in *Econometrica* in 2000, by Darrell Duffie of Stanford University, Jun Pan of Massachusetts Institute of Technology, and Kenneth Singleton of Stanford University. The prize committee chose this paper for its important methodological contribution to the influential literature of affine models. In particular this paper provides (i) an analytical treatment for general affine models where both the discount rate and payoffs are allowed to be affine jump-diffusions of the state variables, and (ii) shows that their analytical treatment permits them to identify an important jumps-in-volatility channel in option prices. These methods have influenced subsequent work in modeling the term-structure of interest rates, defaultable securities, contingent claim prices, and consumption-based asset pricing models. Their methods facilitates assessing the role of jump-diffusions in affine asset pricing models and ultimately help develop better empirical models for asset prices.

Earlier papers by Vasicek (1977) and Cox, Ingersoll, and Ross (1985) present affine term-structure models using Gaussian and square-root diffusions respectively. Duffie and Kan (1998) develop term-structure models of affine diffusions while Dai and Singleton (2000, 2002) provide rigorous empirical evaluation of these models to account for the evolution of term-structure of interest rates. This class of models opened an extensive empirical literature to evaluate the term-structure of interest rates. An important step in developing these models further was to allow for the possibility of default with time-varying intensity in order to price corporate bonds and other credit-sensitive instruments. Duffie-Pan-Singleton (DPS) (2000) provide methods to price these default sensitive instruments as well as other contingent claims. Their methods are also exploited in Duffie and Singleton (1999) to model term structure of defaultable bonds. The DPS paper and its affine predecessors thus paved the way for significant work on the term structure and credit default instruments. Their methods and insights have led to a better understanding of economic channels and

model specifications needed to explain risk premia in bond returns and the level and volatility of yields.

The DPS paper methodology has been very influential. For example, Duffee (2002) studies government yields and the term premia by exploring alternative "essentially affine" specifications of the price of risk. Collin-Dufresne and Goldstein (2002) use the DPS methodology to study incompleteness in the bond market in the form of unspanned stochastic volatility. Longstaff, Mithal, and Neis (2005) use insights from DPS and study corporate yield spreads. Hansen and Scheinkman (2009) utilize the affine-jump specification of DPS for cashflows as an example in their analysis of long-run risk return tradeoffs for nonlinear continuous-time Markov environments. Eraker, Johannes and Polson (2003) use the DPS methods to empirically document that jumps in both returns and return volatility are important in explaining option prices. Duffie, Pederson, and Singleton (2003) use the DPS framework to study defaultable sovereign bonds. In the context of consumption-based models Eraker and Shaliastovich (2008), Wachter (2013) and Dreschler (2013) exploit DPS methods to characterize the equilibrium and asset prices.

Given the extensive use of affine models and their generalizations in models of asset prices, the DPS approach will continue to be highly relevant to future work related to asset prices.

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